Broadcasters Competition on Quality: a Welfare Perspective

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Abstract

The present paper provides a vertical differentiated model of a broadcasting market with a two-sided approach. We calculate the equilibrium in terms of advertising levels, subscription fees and qualities provision, both in monopoly and in duopoly where the market is uncovered. Furthermore, welfare considerations are made for all market structure by considering viewers' surplus.

Key words: two-sided market, broadcasting, quality, welfare.

JEL codes: D42, D43, L15, L82

1 Introduction

Television broadcasting has a long history of salient regulation problems which have been recently emphasized by the convergence among Internet, computer software and telecommunications. Conventionally, regulatory issues in media markets are classified into economic and non-economic features. Where the former are related to the structure of the supply side: the definition of the relevant market, the assessment of the degree of concentration and competition, the impact of the ownership structure and the conditions of the access to the broadcasting service. While the latter mainly focus on the broadcasting contents and the control of advertising. (Rowat, 2007). The interplay among these economic and non-economic issues, which is a peculiar feature of broadcasting, deserves a closer attention from policy makers and antitrust authorities.

In this respect, our paper would provide a unified framework to deal with all these issues and to analyze broadcasting competition with particular concern on quality, prices, share of audience and consumers' surplus. More precisely, it analyzes the role of competition in a two-sided market characterized by vertical differentiation.

Quality is a first ingredient of our model. Despite the fact that quality is a relevant characteristic of broadcasting market, it lacks a clear and common economic definition (Born and Prosser, 2001). At a first glance quality could be associated with technological innovations that have deeply affected broadcasting, such as higher-definition images or interactive services. In this perspective, quality of broadcasting can be interpreted in the standard vertical approach. However if we focus on content, quality is more complex to be defined. As matter of facts, content's quality can be related to accuracy, truth, impartiality and immediacy of information that helps form public opinion, expresses minority voices and performs a watchdog role for the public interest.[1] For instance, Collins (2007), in the public service broadcasting debate, associates quality to the purpose of providing not only entertainment but also education, learning and cultural excellence without ignoring niche interests.[2] However it is worth to notice that viewers' perception might differ concerning these dimensions. Indeed, audience has also a taste for variety of broadcasting output, including cultural programme, popular genres, and sport events. Therefore an increase in content quality does not necessarily translate into an upward shift of demand and audience. Hence, some dimensions of content's quality may encompass an horizontal feature.

Nevertheless, in a specific genre, all viewers prefer high quality contents rather than low quality contents,which implies vertical competition on the market. Given these consideration, in the present paper we assume that broadcasters provide vertically differentiated output with respect to quality.

A second important aspect we would like to address is the role of competition in a two-sided market. The reason to consider this kind of market structure is that .broadcasting networks compete on two sides, namely, audience and advertisers, in order to maximize profits. Advertising is typically considered a nuisance to the audience and it represents a negative externality, while the audience exerts a positive externality for advertisers. Therefore, competition has a broader meaning with respect to the standard industrial organization and might generate different results and policy implications. In our setup, viewers are single-homing, while advertisers are multi-homing, meaning that platforms have monopoly power in providing access to their single-homing customers. In this respect, platforms act as "bottlenecks"

[1] Mepham (1990) argues that there is a general rule for assessing television quality, and that is 'whether or not [its production] is governed by an ethic of truth-telling'.

[2] Ellmann (2014) distinguishes between "soft" and "hard" attributes to media consumption. He defines "hard" or informative attributes of media as those which generate positive social externalities, while"soft" attributes are those with only private value, such as graphic quality, sensationalism and entertainment.

between advertisers and consumers by offering sole access to their respective set of consumers. This assumption is crucial to explain the prevailing competition on the consumer side. [3] We also model advertisers as non-strategic: their payoffs do not depend on what other advertisers do but rather on an advertising benefit related to market demand. This behavior suits the case of informative advertising.

Finally, is well known that media markets are characterized by a broad range of business models, both under private and public ownership:[4] free-to-air TV, where broadcast platforms are only financed through advertising revenues, pay TV, where broadcast stations are financed through subscription revenues, and a mixed regime,where broadcast platforms are financed through both subscription fees and advertising. We consider a very general framework in which platforms are financed both by advertising and subscription fees. [5]

As previously mentioned, we provide a model of platforms competition in a framework of vertical differentiation. In a context where platforms endogenously provide quality levels, we calculate the equilibrium values of advertising, the optimal subscription fees for viewers and the provision of quality. In particular, we take into account a single-channel and multi-channel monopoly,as well as a duopoly. In our analysis, we want to stress the importance of having a market which is never covered *ex-ante*. We believe indeed that the potential demand has a relevant role and might shake the equilibrium configuration, in terms of price, quality, audience size and advertising. Furthermore, the uncovered market configuration fits very well the case of broadcasting market, which is characterized by continuos technological turmoil with the creation of new market segments. We also calculate the consumers' surplus for each market configuration to figure out whether the interplay among contents quality, subscription fees and advertising might benefit audience.

To anticipate the results, we show that viewers are always better off when they are free to choose among channels of different qualities. In our two-sided framework with endogenous quality provision, there are two forces at stake. Higher quality induces consumers to pay higher subscription fees to join the platform. In turn, the platform can extract a surplus on the advertiser side and "invest" them in a reduction of subscription fees, implying that advertisers cross-subsidize single-homing consumers.

[3] For a further discussion on the role of the single-homing or multi-homing assumption, see Roger (2010).

[4] In Italy, for instance, there exists a public broadcaster financed by subscription fees (canone RAI) as well as advertising revenues. At the same time, there exist both free-to-air private operators such as Mediaset that are totally financed through advertising and private pay-TV providers financed through subscription fees and advertising revenues (e.g., Sky).

[5] Subscription fees are set in general terms and could be both positive or negative, encompassing the possibility of subsidization.

Therefore a sort of substitution between quality and advertising arises. We also show that competition is beneficial for the audience, resulting in a viewers' surplus which is larger in the duopoly configuration then in the monopoly, even when both provide high and low-quality channels. Finally, we illustrate that the chance of catching extra viewers, as the uncovered market share, discipline the platforms' behavior in duopoly making consumers' surplus higher.

1.1 Related Literature

Our paper belongs to the literature of vertically differentiated two-sided markets dealing with welfare issues. In this stream, Armstrong (2006) and Weeds (2013) provide a model with endogenous quality provision in the two-sided context of digital broadcasters. By comparing competition in two different regimes, free-to-air and pay TV, they show that program quality is higher for pay TV, which is also optimal from a social point of view. In a similar setting, Anderson (2007) analyzes the effect of an advertising cap on the quality provision and the welfare change of a monopoly broadcaster. He shows that advertising time restrictions may improve welfare but may decrease program quality.[6] Kind et al. (2007) perform a welfare analysis with endogenous quality provision and find that a merger between TV channels may be welfare improving. More recently, Lin (2011) extended the analysis to direct competition between different business model, where one platform operates as a free-to-air TV while the second as a pay-TV. In this framework, he shows that platforms vertically differentiate their programs according to the degree of viewers' dislike of advertising. In the same approach, Gonzales-Mestre and Martinez-Sanchez (2013) study how public-owned platforms affects the program quality provision, the social welfare and the optimal level of advertising. Differently, from our model, all the above contributions focus on the duopoly case, neglecting monopoly behavior with the exception of Anderson (2007). Furthermore, duopoly setting is always assumed to be covered, preventing any welfare consideration about the role of increasing demand. Conversely, we relax this assumption introducing a set up of uncovered market. We also provide a comparison between the uncovered and the covered market structure from a welfare perspective.

The paper is organized as follows. Section 2 illustrates the case of a multi-channel monopoly broadcaster: set-up and equilibrium. Then, Section 3 focuses on the welfare comparison between the multi-channel monopoly broadcaster and a single-channel one. Then , Section 4 introduces competition among broadcasters: set-up and equilibrium, while Section 5 deals with the welfare effects Finally we

[6]Without a specific reference to quality provision, Dukes (2004) and Anderson and Coate (2005) show that monopoly media ownership may increase welfare.

provide some conclusions in Section 6.

2 The Multi-Channel Broadcaster

For the sake of exposition we describe first the case of a multi-product monopoly platform and second the duopoly case. [7]

A multiproduct monopoly platform can provide vertical differentiated *channels* to a uniform distribution of individuals (viewers[8]) of mass 1. We refer to this platform as a multi-channel broadcaster.

Individuals of mass 1 are assumed to be single-homing. The utility of an individual accessing platform's channel i is:

$$u_i = V - \delta a_i + \beta\theta_i - s_i \tag{1}$$

and zero otherwise. The channel i's quality is denoted by the parameter θ_i which belongs to a technological range $\Theta = [\underline{\theta}, \overline{\theta}]$. For now, the only restrictions on this set are $\underline{\theta} > 0$ and $\underline{\theta} < \overline{\theta}$. Individuals have a private valuation for information , expressed by the parameter $\beta \sim U[0, 1]$. Moreover, they incur in a nuisance cost δa_i [9] due to the presence of advertising on the channels. Finally s_i stands for the subscription charge. These fees are set in general terms and ex-ante they can be positive or negative. We can interpret a negative subscription fee as a subside. [10]

If the platform provides two channels of different quality, θ_H and θ_L (with $\theta_H > \theta_L$) it obtains the following audience shares (for each channel):

$$B_H = 1 - \beta_{HL} = 1 - \frac{\delta\left(a_H - a_L\right)}{\left(\theta_H - \theta_L\right)} - \frac{\left(s_H - s_L\right)}{\left(\theta_H - \theta_L\right)} \tag{2}$$

$$B_L = \beta_{HL} - \beta_{0L} = \frac{\delta\left(a_H - a_L\right)}{\left(\theta_H - \theta_L\right)} + \frac{\left(s_H - s_L\right)}{\left(\theta_H - \theta_L\right)} - \left(\frac{\delta a_L - V}{\theta_L} - \frac{s_L}{\theta_L}\right) \tag{3}$$

where β_{HL} and β_{0L} characterize respectively the individual indifferent between the two options and the one indifferent between accessing the low channel or not accessing at all.

[7] The model in this section builds on that of Battaggion and Drufuca (2014).

[8] We consider a broadcasting market, which well fits our setting. However, in principle this model might be refered also to a broader range media (newspapers, as example).

[9] This cost depends on the intensity of advertising on the channel a_i and on a parameter of viewers' aversion to ads δ. This parameter is assumed to be invariant across individuals.

[10] Subsidisation is not uncommon in media markets. Consider, as an example, the case of newspapers.

Advertisers are producers of mass 1 who access the platform to advertise their products to individuals. They sell products of quality α that are produced at constant marginal costs set equal to zero. Product quality α is distributed on an interval $[0, 1]$ according to a distribution function $F(\alpha)$ [11]. Individuals have a willingness to pay α for a good of quality α. Each producer has monopoly power and can therefore extract the full surplus from individuals by selling its product at price equal to α. As standard in this class of models, we assume advertising to be *informative* and that consumers watching an advertisement always buy the good advertised. Hence, we refer to producers as advertisers. Differently from viewers, advertisers are allowed to multi-home. Advertisers have to pay an advertising charge r_i endogenously determined for each channel Due to the assumption of single homing on the viewers' side, each channel behaves as a "monopoly" in carrying its audience to advertisers. Therefore, r_i is set by the platform in order to leave the marginal advertiser with zero profit:

$$\alpha_i = \frac{r_i}{NB_i} \tag{4}$$

Thus, the amount of advertising for each channel is the share of advertisers with $\alpha > \alpha_i$:

$$a_H = 1 - F\left(\frac{r_H}{B_H}\right) \tag{5}$$

$$a_L = 1 - F\left(\frac{r_L}{B_L}\right) \tag{6}$$

The platform sets advertising spaces and subscription prices (unconstrained) and it can provide its channels' quality θ_H and θ_L by incurring a fixed cost K.[12] In other words, once the cost is incurred, the higher-quality outlet can be provided to individuals without any additional charges. Notice that since our costs are also fixed in quantity, they meet the requirement of costs' separability. Hence, the multi-channel media platform collects revenues from individuals and advertisers on both channels:

$$\Pi_{MP} = (B_H s_H + B_L s_L) + a_H r_H + a_L r_L - 2K \tag{7}$$

[11] In the discussion of our results, we will consider also the special case of a uniform distribution of advertisers.

[12] This assumption fits very well the structure of the ICT and media markets, where there is a prominent role of fixed costs compared to marginal ones (see e.g., Shapiro and Varian (1998), Areeda and Hovenkamp (2014))

According to the literature, we define the advertising revenues per individual as $\rho(a_i)$:

$$\rho(a_i) = \frac{a_i r_i}{B_i} = \frac{a_i F^{-1}(1-a_i)NB_i}{B_i} = a_i F^{-1}(1-a_i) \tag{8}$$

We assume $\rho(a_i)$ to be concave in the interval $a \in [0,1]$. Given that $\rho(a_i) = 0$ for $a_i = 0$ and $a_i = 1$, the function is single-peaked. Hence, profits rewrite as follow:

$$\Pi_{MP} = B_H(s_H + \rho(a_H)) + B_L(s_L + \rho(a_L)) - 2K \tag{9}$$

We assume a three-stage game. First the monopoly platform chooses the levels of quality. Second, it sets subscription fees and advertising spaces Finally, in the third stage, viewers and advertisers simultaneously decide whether to join a channel.

2.1 Subscription Fees and Advertising Intensities

Having defined the demand function of viewers and advertisers, for given prices we solve the game backwards, from stage three. This determines how advertising charges react to pay-per-view prices s_i and to

advertising levels a_i:

$$r_H\left(s_H, s_L, a_H, a_L, \theta_H, \theta_L\right) = F^{-1}(1-a_H)\left(\frac{\theta_H - \theta_L - (s_H - s_L) - \delta(a_H - a_L)}{\theta_H - \theta_L}\right) \tag{10}$$

$$r_L\left(s_H, s_L, a_H, a_L, \theta_H, \theta_L\right) = F^{-1}(1-a_L)\left(\tfrac{(s_H - s_L) + \delta(a_H - a_L)}{\theta_H - \theta_L} - \tfrac{s_L + \delta a_L - V}{\theta_L}\right) \tag{11}$$

The commercial multi-channel broadcaster relies on advertising revenues and subscription fees to fund its services.

$$\begin{cases} \max\limits_{a_H, a_L, s_H, s_L} \Pi_{MP} = \pi_L + \pi_H = B_H(s_H + \rho(a_H)) + B_L(s_L + \rho(a_L)) - 2K \\ s.t. a_H \geq 0 \\ a_L \geq 0 \end{cases}$$

The platform maximizes profits (9), with respect to advertising intensity (a_H, a_L) and subscription fees (s_H, s_L) for each channel, subject to a positivity constraint on advertising. The following Proposition summarizes results regarding advertising.

Proposition 1 *The multi-channel monopoly broadcaster chooses the same advertising intensity, independently of quality and subscription revenues*

$$\rho'(a_i) = \delta$$

for $i = H, L$.

Proof. First order conditions with respect to the advertising spaces and subscription fees are respectively, for $i, j = H, L$ with $i \neq j$:

$$\frac{\partial \pi_{MP}}{\partial s_i} = \frac{\partial B_i}{\partial s_i}(s_i + \rho_i) + B_i(1 + \frac{\partial \rho_i}{\partial s_i}) + \frac{\partial B_j}{\partial s_i}(s_j + \rho_j) + B_j(\frac{\partial \rho_j}{\partial s_i}) = 0$$
$$\frac{\partial \pi_{MP}}{\partial a_i} = \frac{\partial B_i}{\partial a_i}(s_i + \rho_i) + B_i(\frac{\partial s_i}{\partial a_i} + \frac{\partial \rho_i}{\partial a_i}) + \frac{\partial B_j}{\partial a_i}(s_j + \rho_j) + B_j(\frac{\partial s_j}{\partial a_i} + \frac{\partial \rho_j}{\partial a_i}) \leqq 0$$

Given the construction of advertising revenues per individual (see equation (8)), we have that $\frac{\partial \rho_i}{\partial a_j} = 0$. Moreover $\frac{\partial s_i}{\partial a_i} = 0$ and $\frac{\partial s_i}{\partial a_j} = 0$. Hence, first order conditions simplify as follows:

$$\frac{\partial \pi_{MP}}{\partial s_i} = \frac{\partial B_i}{\partial s_i}(s_i + \rho_i) + B_i(1 + \frac{\partial \rho_i}{\partial s_i}) + \frac{\partial B_j}{\partial s_i}(s_j + \rho_j) + B_j(\frac{\partial \rho_j}{\partial s_i}) = 0$$
$$\frac{\partial \pi_{MP}}{\partial a_i} = \frac{\partial B_i}{\partial a_i}(s_i + \rho_i) + B_i(\frac{\partial \rho_i}{\partial a_i}) + \frac{\partial B_j}{\partial a_i}(s_j + \rho_j) \leqq 0$$

It is easy to show that $\frac{\partial B_H}{\partial a_H} = \delta \frac{\partial B_H}{\partial s_H}$, $\frac{\partial B_L}{\partial a_L} = \delta \frac{\partial B_L}{\partial s_L}, \frac{\partial B_H}{\partial a_L} = \delta \frac{\partial B_H}{\partial s_L}$ and $\frac{\partial B_L}{\partial a_H} = \delta \frac{\partial B_L}{\partial s_H}$. FOC rewrites as follows

$$\frac{\partial \pi_{MP}}{\partial s_H} = \frac{\partial B_H}{\partial s_H}(s_H + \rho_H) + B_H + \frac{\partial B_L}{\partial s_H}(s_L + \rho_L) = 0 \tag{12}$$
$$\frac{\partial \pi_{MP}}{\partial s_L} = \frac{\partial B_L}{\partial s_L}(s_L + \rho_L) + B_L + \frac{\partial B_H}{\partial s_L}(s_H + \rho_H) = 0 \tag{13}$$
$$\frac{\partial \pi_{MP}}{\partial a_H} = \delta \frac{\partial B_H}{\partial s_H}(s_H + \rho_H) + B_H \rho'_H + \delta \frac{\partial B_L}{\partial s_H}(s_L + \rho_L) \leqq 0 \tag{14}$$
$$\frac{\partial \pi_{MP}}{\partial a_L} = \delta \frac{\partial B_L}{\partial s_L}(s_L + \rho_L) + B_L \rho'_L + \delta \frac{\partial B_H}{\partial s_L}(s_H + \rho_H) \leqq 0 \tag{15}$$

By substitution we get from (14) and (15)

$$B_H \rho'_H + \delta(-B_H) \leqq 0$$
$$B_L \rho'_L + \delta(-B_L) \leqq 0$$

If $a_i > 0$ for $i = H, L$, then

$$\rho'(a_H^*) = \delta \tag{16}$$
$$\rho'(a_L^*) = \delta \tag{17}$$

■

According to the above Proposition an optimal strategy is to set a fixed advertising space for each channel just depending on the disutility of the viewers. Moreover, the multi-channel broadcaster does not set the maximum intensity of advertising or the amount that maximize revenues per viewer, i.e. $\rho'(a_i) = 0$. This result is in line with the literature dealing with the issue of competitive bottlenecks on the audience side. From optimality conditions (12) and (13), given a_H^* and a_L^*, we obtain equilibrium subscription fees, s_H^* and s_L^* , and shares on viewers' side. B_H^* and B_L^*, as function of quality, revenues per viewer and advertising level:

$$s_H^* = \frac{\theta_H + V - a^*\delta - \rho(a^*)}{2} \tag{18}$$

$$s_L^* = \frac{\theta_L + V - a^*\delta - \rho(a^*)}{2} \tag{19}$$

$$B_H^* = \frac{1}{2} \tag{20}$$

$$B_L^* = \frac{1}{2} - \left(\frac{\theta_L - V + \delta a^* - \rho(a^*)}{2\theta_L}\right) \tag{21}$$

The above values show a profit neutrality result, where revenues from the advertising side are counterbalanced by a decrease on the subscription fees, irrespective of the channel. Moreover, given that subscription fees positively depend on quality, a sort of substitutability between advertising and quality emerges. It is relevant to notice that the high quality channel always covers half of the viewers' market, while the audience of the low quality channel relies on quality, fees and advertising. If the monopoly would cover the whole market, it equally splits the audience between the two channels. Otherwise the low quality channel has always less viewers.

Notice that advertising revenues $\rho(a^*)$ depend on the distribution function of advertisers. In this respect, we can get a sharper intuition of our results by assuming a specific type of distribution. In particular we consider the case of a uniform distribution of advertisers, obtaining the following equilibrium values:

$$a_H^* = a_L^* = a^* = \frac{1 - \delta}{2} \tag{22}$$

$$s_H^* = \frac{\theta_H + V - \frac{(1-\delta)(1+3\delta)}{4}}{2}$$

$$s_L^* = \frac{\theta_L + V - \frac{(1-\delta)(1+3\delta)}{4}}{2}$$

$$B_H^* = \frac{1}{2}$$
$$B_L^* = \frac{1}{2} - \frac{\theta_L - V - (\frac{1-\delta}{2})^2}{2\theta_L}$$

In the uniform case equilibrium fees and advertising intensity just depend on quality and disutility from advertising δ.

2.2 Quality

At stage 1, the multichannel platform chooses quality levels.[13] Its profits are:

$$\pi_{MP} = \frac{\theta_H}{4} + \frac{(V + \rho(a^*) - \delta a^*)(2\theta_L + V + \rho(a^*) - \delta a^*)}{4\theta_L} - 2K$$

Looking at first order conditions we get

$$\frac{\partial \pi_{MP}}{\partial \theta_H} = \frac{1}{4} > 0 \tag{23}$$
$$\frac{\partial \pi_{MP}}{\partial \theta_L} = -\frac{1}{4\theta_L^2} \left(V + \rho(a^*) - \delta a^*\right)^2 < 0 \tag{24}$$

Hence we get a result of maximal differentiation, as stated in the following Proposition

Proposition 2 *When viewers differ in willingness to pay for quality, the multichannel broadcaster chooses to maximally differentiate on quality: given a technological constraint it chooses the minimal quality for the L channel while it sets the highest quality for the H one.*

$$\theta_H^* = \overline{\theta}$$
$$\theta_L^* = \underline{\theta}$$

Moreover, it charges different subscription fees for the two channels, according to the quality level:

$$s_H^*(\overline{\theta}) > s_L^*(\underline{\theta})$$

[13] The result of this stage follows the assumption of fixed cost of quality, K. However, we obtain similar outcomes different functional form for the quality cost, (see discussion in Appendix 7.1).

According to the above Proposition 2, the profit becomes:

$$\pi^*_{MP} = \frac{\overline{\theta}}{4} + \frac{(V - \delta a^* + \rho(a^*))(2\underline{\theta} + V - \delta a^* + \rho(a^*))}{4\underline{\theta}} - 2K \tag{25}$$

In the uniform case, equilibrium values are

$$a^*_H = a^*_L = a^* = \frac{1-\delta}{2}$$

$$s^*_H = \frac{\overline{\theta} + V - \frac{(1-\delta)(1+3\delta)}{4}}{2}$$
$$s^*_L = \frac{\underline{\theta} + V - \frac{(1-\delta)(1+3\delta)}{4}}{2}$$

$$B^*_H = \frac{1}{2}$$
$$B^*_L = \frac{1}{2} - \frac{\underline{\theta} - V - (\frac{1-\delta}{2})^2}{2\underline{\theta}}$$

$$\pi^*_{MP} = \frac{\overline{\theta}}{4} + \frac{(V + (\frac{1-\delta}{2})^2)(2\underline{\theta} + V + (\frac{1-\delta}{2})^2)}{4\underline{\theta}} - 2K$$

2.3 Viewers' Surplus

We turn now to the welfare implications. Let us start by considering the general formulation of the viewers' surplus:

$$\begin{aligned} SC_{MP} &= \int_0^{\beta_{0L}} (u_0)\, d\beta + \int_{\beta_{0L}}^{\beta_{LH}} (u_L)\, d\beta + \int_{\beta_{LH}}^{1} (u_H)\, d\beta \\ &= \tfrac{1}{2}\beta^2_{LH}\theta_L + \beta_{LH}\left(V - \delta a_L - s_L\right) - \tfrac{1}{2}\beta^2_{0L}\theta_L - \beta_{0L}\left(V - \delta a_L - s_L\right) \\ &\quad + \tfrac{1}{2}\left(1 - \beta^2_{LH}\right)\theta_H + \left(1 - \beta_{LH}\right)\left(V - s_H - \delta a_H\right) \end{aligned} \tag{26}$$

By substituting equilibrium values, we get:

$$SC^*_{MP} = \frac{\overline{\theta}}{8} + \frac{(2\underline{\theta} + V + \rho(a^*) - \delta a^*)(V + \rho(a^*) - \delta a^*)}{8\underline{\theta}} \tag{27}$$

In the uniform case, provided that $a^* = \left(\frac{1-\delta}{2}\right)$, equation (27) rewrites as follows:

$$SC_{MP} = \frac{\overline{\theta}}{8} + \frac{(2\underline{\theta} + V + (\frac{1-\delta}{2})^2)(V + (\frac{1-\delta}{2})^2)}{8\underline{\theta}}$$

which helps in assessing the effects of the nuisance parameter δ and technological range $\Theta = (\underline{\theta}, \overline{\theta})$. The disutility parameter affects the consumers' surplus in two ways. First, an increase in δ has a direct negative impact on the individual utility, for a given advertising intensity. Second, there is an indirect impact through advertising. Indeed, in equilibrium, an increase in δ reduces advertising intensity. In turn, the effect of a lower advertising is twofold; the advertising cost per viewer δa^* drops back and advertising revenues per viewers $\rho(a^*)$ are reduced. This latter effect induces a higher subscription fees due to profit-neutrality. The direct effect and the indirect one on subscription fees prevail inducing a negative impact on surplus.

$$\frac{\partial SC^*_{MP}}{\partial \delta} = \frac{1}{32\underline{\theta}} (\delta - 1) \left(\delta^2 - 2\delta + 4\underline{\theta} + 4V + 1\right) \leq 0$$

For, $\delta < 1$ the above effect is strictly negative, while for $\delta > 1$ the effect is null due to the fact that the platform does not broadcast advertising in any channel.

$$\frac{\partial SC^*_{MP}}{\partial \overline{\theta}} = \frac{1}{8} > 0$$

$$\frac{\partial SC^*_{MP}}{\partial \underline{\theta}} = -\frac{1}{128\underline{\theta}^2} \left(\delta^2 - 2\delta + 4V + 1\right)^2 < 0$$

The above derivatives explain the positive effect of enlarging the technological range. Consumers benefit by widening differentiation between the two channels.

3 Multi-Channel vs Single-Channel Broadcaster

In order to asses the welfare analysis it should be relevant to compare our previous insights with case of a single-channel monopoly broadcaster. The derivation of the equilibrium for the single-channel monopoly is along the line of the previous Subsections. As the structure of the analysis does not vary, the mathematical analysis of this case can be found in the Appendix 7.2.[14] The results provide an equal ground for comparing the multi-channel case to the single-channel case. The results for the single-channel case are summarized in the following Proposition:

[14]The results for the single-channel case rely on our previous paper Battaggion and Drufuca (2014).

Proposition 3 *A single-channel monopoly platform which maximizes profit in an uncovered market, shows the following equilibrium levels of advertising, subscription fees and audience share:*

$$a_M^* = \frac{1-\delta}{2}$$

$$s_M^* = \frac{V + \theta_M^* - \rho(a_M^*) - \delta a_M^*}{2}$$

$$B_M^* = \frac{V + \theta_M^* + \rho(a^*) - \delta a^*}{2\theta_M^*}$$

Moreover, regarding quality, two possible equilibrium configurations emerge, depending on the technological range

- $\theta_M^* = \overline{\theta}$ *if* $\Theta_{RL} = \left[\underline{\theta}, \overline{\theta}\right]$ *with* $\underline{\theta} = \rho(a) - \delta a$
- $\theta_M^* = \underline{\theta}$ *if* $\Theta_{RH} = \left[\underline{\theta}, \overline{\theta}\right]$ *with* $\underline{\theta} > 0$ *and* $\overline{\theta} = \rho(a) - \delta a$

Proof. See Appendix 7.2 ■

We proceed by comparing viewers' surplus, subscription fees and audience shares in the multi-channel case and the single one.

Proposition 4 *In the multi-channel monopoly viewers' surplus is larger than in the single-channel case, independently of the technological range of quality.*

Proof. We consider first the case of $\Theta_{RL} = \left[\underline{\theta}, \overline{\theta}\right]$ with $\underline{\theta} = \rho(a) - \delta a$. We first compare the multi-channel platform with the single-channel platform that provides the maximum quality. Viewers' surplus are respectively:

$$SC_{MP}^*(\underline{\theta}, \overline{\theta}) = \frac{\overline{\theta}}{8} + \frac{(2\underline{\theta} + V - \delta a^* + \rho(a^*))(V - \delta a^* + \rho(a^*))}{8\underline{\theta}}$$

$$SC_M^*(\overline{\theta}) = \frac{1}{8\overline{\theta}}\left(V + \rho(a^*) - \delta a^* + \overline{\theta}\right)^2$$

If $\overline{\theta} > \underline{\theta}$

$$SC_{MP}^*(\underline{\theta}, \overline{\theta}) - SC_M^*(\overline{\theta}) = \frac{(\overline{\theta} - \underline{\theta})(V - \delta a^* + \rho(a^*))^2}{8\overline{\theta}\underline{\theta}} > 0$$

Analogously, in the case $\Theta_{RH} = \left[\underline{\theta}, \overline{\theta}\right]$ with $\underline{\theta} > 0$ and $\overline{\theta} = \rho(a) - \delta a$, we obtain the same result:

$$SC_{MP}^*(\underline{\theta}, \overline{\theta}) - SC_M^*(\underline{\theta}) = \frac{\overline{\theta} - \underline{\theta}}{8} > 0$$

■

According to Proposition 4, the multi-channel monopoly is welfare improving, for what concerns viewers, with respect to the single-channel one, independently of the technological range of quality. At a first glance it seems that viewers benefit from the presence of multiple channels of different quality. In order to figure out the driving forces of this result, we compare equilibrium audiences and subscription fees. We make this comparison for two cases, either the single-channel monopoly choosing $\bar{\theta}$ or the single-channel monopoly choosing $\underline{\theta}$. [15]

Table 1: COMPARISON AMONG REGIMES

	Case Θ_{RL}	Case Θ_{RH}
Viewers' Surplus	$SC^*_{MP} > SC^*_M$	$SC^*_{MP} > SC^*_M$
Channels' Quality Levels	$\theta^*_H = \theta^*_M$ $\theta^*_L < \theta^*_M$	$\theta^*_H > \theta^*_M$ $\theta^*_L = \theta^*_M$
Viewers' Fees	$s^*_M = s^*_H$ $s^*_M > s^*_L$	$s^*_M < s^*_H$ $s^*_M = s^*_L$
Viewers' Market Shares	$B^*_{MP}(\underline{\theta}, \bar{\theta}) > B^*_M(\bar{\theta})$	$B^*_{MP}(\underline{\theta}, \bar{\theta}) = B^*_M(\underline{\theta})$
Advertisers' Market Shares	$a^*_H = a^*_L = a^*_M$	$a^*_H = a^*_L = a^*_M$

Note: In this table, we compare equilibrium values of the multichannel monopoly broadcaster and the single-channel one. The case with the single-channel choosing maximum quality (Case Θ_{RL}) is
shown in the first column, the case with minimum quality (Case Θ_{RH}) in the second column.

In the first case we disentangle two effects: one on subscription fees and the other on audience's share. [16] The multi-channel broadcaster serves a larger market share of viewers with respect to the single-channel monopoly. Moreover it charges a lower price on the low quality channel. Hence, in this case, the welfare improving effect is driven by prices and market shares.

[15]These two cases are set by considering the appropriate restrictions on the technological range of quality.

[16]Provided that $V + \rho(a^*) - \delta a^* > 0$.

Similarly, we compare subscription fees and the audience's share for the second case: we can state that viewers benefit from the possibility of a multi-channel choice with a high quality option. Whereas, there is no positive effect on fees and share. Again, to highlight our findings, we illustrate our results in the case of a uniform distribution of advertisers, as summarized in the following Remark

Remark 5 *We consider the case of a uniform distribution of advertisers. We show that viewers' surplus is higher if they are served by a multi-channel monopoly compared to a single-channel one. This result holds independently of the technological range of quality; that is, either if the single-channel chooses the minimum quality (Θ_{RL}) or it chooses the maximum quality (Θ_{RH}). For what concern prices and audience's shares, we obtain the following equilibrium values, which confirm our previous insights on the different effects driving our results on surplus.*

Table 2: EQUILIBRIUM VALUES (Uniform Case)

	Case Θ_{RL}	Case Θ_{RH}
Channels' Quality Levels	$\theta^*_H=\overline{\theta}$ $\theta^*_L=\underline{\theta}=\frac{(1-\delta)^2}{4}$ $\theta^*_M=\overline{\theta}$	$\theta^*_H=\overline{\theta}=\frac{(1-\delta)^2}{4}$ $\theta^*_L=\underline{\theta}$ $\theta^*_M=\underline{\theta}$
Viewers' Fees	$s^*_H=\frac{\overline{\theta}+V-\frac{(1-\delta)(1+3\delta)}{4}}{2}$ $s^*_L=\frac{\underline{\theta}+V-\frac{(1-\delta)(1+3\delta)}{4}}{2}$ $s^*_M=\frac{V+\overline{\theta}-\left(\frac{1-\delta}{2}\right)\left(\frac{1+3\delta}{2}\right)}{2}$	$s^*_H=\frac{\overline{\theta}+V-\frac{(1-\delta)(1+3\delta)}{4}}{2}$ $s^*_L=\frac{\underline{\theta}+V-\frac{(1-\delta)(1+3\delta)}{4}}{2}$ $s^*_M=\frac{V+\underline{\theta}-\frac{(1-\delta)(1+3\delta)}{4}}{2}$
Viewers' Market Shares	$B^*_{MP}(\underline{\theta},\overline{\theta})=\frac{V+\underline{\theta}+\frac{(1-\delta)^2}{4}}{2\underline{\theta}}$ $B^*_M(\overline{\theta})=\frac{V+\overline{\theta}+\frac{(1-\delta)^2}{4}}{2\overline{\theta}}$	$B^*_{MP}(\underline{\theta},\overline{\theta})=\frac{V+\underline{\theta}+\frac{(1-\delta)^2}{4}}{2\underline{\theta}}$ $B^*_M(\underline{\theta})=\frac{V+\underline{\theta}+\frac{(1-\delta)^2}{4}}{2\underline{\theta}}$
Advertisers' Market Shares	$a^*_H=a^*_L=a^*_M=\frac{1-\delta}{2}$	$a^*_H=a^*_L=a^*_M=\frac{1-\delta}{2}$

Proof. For what concerns consumers' surplus, if $\Theta_{RL}=\left[\underline{\theta},\overline{\theta}\right]$ with $\underline{\theta}=\rho(a-\delta a)$:

$$SC^*_{MP}(\underline{\theta},\overline{\theta})=\frac{\overline{\theta}}{8}+\frac{(2\underline{\theta}+V+(\frac{1-\delta}{2})^2)(V+(\frac{1-\delta}{2})^2)}{8\underline{\theta}}$$

$$SC^*_M(\overline{\theta})=\frac{1}{8\overline{\theta}}\left(V+(\frac{1-\delta}{2})^2+\overline{\theta}\right)^2$$

Then if if $\overline{\theta} > \underline{\theta}$:

$$SC^*_{MP}(\underline{\theta}, \bar{\theta}) - SC^*_M(\overline{\theta}) = \frac{(\overline{\theta} - \underline{\theta})(V + (\frac{1-\delta}{2})^2)^2}{8\overline{\theta}\underline{\theta}} > 0$$

If $\Theta_{RH} = \left[\underline{\theta}, \overline{\theta}\right]$ with $\underline{\theta} > 0$ and $\overline{\theta} = \rho(a - \delta a)$:

$$SC^*_{MP}(\underline{\theta}, \bar{\theta}) = \frac{\overline{\theta}}{8} + \frac{(2\underline{\theta} + V + (\frac{1-\delta}{2})^2)(V + (\frac{1-\delta}{2})^2)}{8\underline{\theta}}$$

$$SC^*_M(\underline{\theta}) = \frac{1}{8\underline{\theta}} \left(V + (\frac{1-\delta}{2})^2 + \underline{\theta} \right)^2$$

Then if if $\overline{\theta} > \underline{\theta}$:

$$SC^*_{MP}(\underline{\theta}, \bar{\theta}) - SC^*_M(\underline{\theta}) = \frac{\overline{\theta} - \underline{\theta}}{8} > 0$$

■

4 Competition among Single-Channel Broadcasters

In this section we modify our set up by considering competition.among broadcasters. We present the case of a duopoly market where two single-channel platforms compete on viewers and advertisers, namely $i = 1, 2$. Without loss of generality we assume that $i = 1$ is the low quality platform, while $i = 2$ is the high quality one. Thus we set $i = L, H$. [17] For the remaining we maintain the same assumptions as in multi-channel set up.

Notice that, differently from the ongoing literature on vertically differentiated media, we consider an ex-ante uncovered market. This framework further complicates the model from an analytical point of view, arising multiple equilibria. To overcome this issue,we restrict the analysis to a local equilibrium: we identify a technological range of qualities allowing a local equilibrium of maximal differentiation to exist. However we strongly believe it is worth to maintain an uncovered set-up since it better point out the effects of competition on audience and prices. In this respect it well fits the features of broadcasting market.

4.1 Viewers' and Advertisers' Shares

We identify two marginal consumers: the one indifferent between not accessing to any platform and accessing the low quality platform

[17] We relax this ex-ante assumption when we look at the choice of quality (stage 1).

$$\beta_{0L} = \frac{s_L + \delta a_L - V}{\theta_L} \tag{28}$$

and the one indifferent between the low quality platform and the high quality one

$$\beta_{LH} = \frac{(s_H - s_L) + \delta(a_H - a_L)}{\theta_H - \theta_L} \tag{29}$$

Given our distribution of the willingness to pay quality, β, the trivial case in which the low-quality platform always faces zero demand in the price game is automatically ruled out. Hence, we consider an ex-ante market structure where both firms are active (meaning that the individuals' demands for both platform H and L are positive).

We do not impose any further condition on the configuration: namely, we consider an ex-ante uncovered duopoly structure. Hence, the high quality platform's share on viewers side is

$$B_H = (1 - \beta_{LH}) = \left(1 - \frac{(s_H - s_L) + \delta(a_H - a_L)}{\theta_H - \theta_L}\right) \tag{30}$$

whereas the low quality platform's share is

$$B_L = (\beta_{LH} - \beta_{0L}) = \left(\frac{(s_H - s_L) + \delta(a_H - a_L)}{\theta_H - \theta_L} - \frac{s_L + \delta a_L - V}{\theta_L}\right) \tag{31}$$

The intensities of advertising for the two platforms are respectively:

$$a_H = 1 - F\left(\frac{r_H}{B_H}\right) \tag{32}$$

$$a_L = 1 - F\left(\frac{r_L}{B_L}\right) \tag{33}$$

Having defined the shares of viewers and of advertisers, for given prices, we solve the game backwards, from stage three, as previously described for the monopoly:

$$r_H\left(s_H, s_L, a_H, a_L, \theta_H, \theta_L\right) = F^{-1}(1 - a_H)\left(\tfrac{\theta_H - \theta_L - (s_H - s_L) - \delta(a_H - a_L)}{\theta_H - \theta_L}\right) \tag{34}$$

$$r_L\left(s_H, s_L, a_H, a_L, \theta_H, \theta_L\right) = F^{-1}(1 - a_L)\left(\tfrac{(s_H - s_L) + \delta(a_H - a_L)}{\theta_H - \theta_L} - \tfrac{s_L + \delta a_L - V}{\theta_L}\right) \tag{35}$$

4.2 Subscription Fees and Advertising Intensities

According to the previous assumptions, each platform maximizes profits subject to a positivity constraint on advertising:

$$\begin{cases} \max_{a_i, s_i} \Pi_i = B_i(s_i + \rho_i) - K \\ \qquad s.t. a_i \geq 0 \end{cases}$$

for $i = H, L$.

Proposition 6 *For each platform $i = H, L$, if the profit maximizing advertising level is positive, then it is constant and it is determined by*

$$\rho'(a_i) = \delta$$

Proof. We consider first the maximization problem of the L platform. Under the assumption that $\frac{\partial B_L}{\partial a_L} = \delta \frac{\partial B_L}{\partial s_L}$ and $\frac{\partial B_H}{\partial a_L} = \delta \frac{\partial B_H}{\partial s_L}$, first order conditions are:

$$\frac{\partial \pi_L}{\partial s_L} = \frac{\partial B_L}{\partial s_L}(s_L + \rho(a_L)) + B_L = 0 \tag{36}$$

$$\frac{\partial \pi_L}{\partial a_L} = \delta \frac{\partial B_L}{\partial s_L}(s_L + \rho(a_L)) + B_L(\rho'(a_L)) \leqq 0 \tag{37}$$

If $a_L > 0$, optimality conditions rewrite as follows

$$\frac{\partial B_L}{\partial s_L}(s_L + \rho(a_L)) = -B_L$$

$$\delta \frac{\partial B_L}{\partial s_L}(s_L + \rho(a_L)) + B_L(\rho\prime(a_L)) = 0$$

Hence, by substitution we get

$$\rho'(a_L) = \delta \tag{38}$$

The same applies to the high quality platform, giving us:

$$\rho'(a_H) = \delta \tag{39}$$

■

The above Proposition states that, for both platforms, a fixed advertising space is the best reply. In particular, the equilibrium intensity of advertising depends just on the nuisance parameter δ. If aversion to ads is "too high", then is optimal to set advertising equal to zero. Hence, the optimal advertising intensity considers just the negative externality of advertisers on viewers, suggesting that both platform just compete on individuals. Indeed, platforms act as "bottlenecks" between advertisers and individuals, by offering sole access to their respective set of individuals.

Moreover, by considering the case of a uniform distribution of advertisers, we point out that:

Remark 7 *We consider the case of a uniform distribution of advertisers. The strategic choices of advertising intensity of the two platform are the same and depend just on the nuisance parameter δ:*

$$a_i^* = \frac{1-\delta}{2} \quad \textit{for } i = H, L$$

if $\delta < 1$. Otherwise, is zero.

We can now compute the subscription fees, the advertising prices and the viewers' shares of the two platforms.

Proposition 8 *At stage 2 platform H and platform L set the following equilibrium values for subscription fees, audience shares and advertising prices:*

$$s_H^* = \frac{(V - a^*\delta + 2\theta_H)(\theta_H - \theta_L) - 3\rho(a^*)\theta_H}{4\theta_H - \theta_L}$$
$$s_L^* = \frac{(2(V - a^*\delta) + \theta_L))(\theta_H - \theta_L) - 2\rho(a^*)\theta_H - \rho(a^*)\theta_L}{4\theta_H - \theta_L}$$

$$B_H^* = \frac{2\theta_H + V + \rho(a^*) - a^*\delta}{4\theta_H - \theta_L}$$
$$B_L^* = \frac{2\theta_H}{\theta_L}\frac{\frac{1}{2}\theta_L + V + \rho(a^*) - a^*\delta}{4\theta_H - \theta_L}$$

$$r_H^* = \frac{\rho(a^*)}{a^*(4\theta_H - \theta_L)}(V + 2\theta_H + \rho(a^*) - a^*\delta)$$
$$r_L^* = \frac{2\rho(a^*)}{a^*(4\theta_H - \theta_L)}(V + \frac{1}{2}\theta_L + \rho(a^*) - a^*\delta)$$

Proof. Given the results of Proposition 6, we compute equilibrium subscription fees for the two platforms from the second FOCs

$$\begin{array}{c} \frac{\partial B_H}{\partial s_H}(s_H+\rho(a_H))+B_H= \\ (-\frac{1}{\theta_H-\theta_L})(s_H+\rho(a_H))+\left(1-\frac{(s_H-s_L)+\delta(a_H-a_L)}{\theta_H-\theta_L}\right)=0 \end{array} \tag{40}$$

$$\begin{array}{c} \frac{\partial B_L}{\partial s_L}(s_L+\rho(a_L))+B_L= \\ (-\frac{1}{\theta_H-\theta_L}-\frac{1}{\theta_L})(s_L+\rho(a_L))+\left(\frac{(s_H-s_L)+\delta(a_H-a_L)}{\theta_H-\theta_L}-\frac{s_L+\delta a_L-V}{\theta_L}\right)=0 \end{array} \tag{41}$$

Since at equilibrium the advertising intensity is the same, $a_i = a$ and $\rho(a_i) = \rho(a)$ for $i = H, L$:

$$s_H = \frac{\theta_H - \theta_L + s_L - \rho(a)}{2} \tag{42}$$

$$s_L = \frac{(V - a\delta)(\theta_H - \theta_L) - \rho(a)\theta_H + \theta_L s_H}{2\theta_H} \tag{43}$$

Then, if $\theta_H > \theta_L > 0$

$$s_H^* = \frac{(V - a^*\delta + 2\theta_H)(\theta_H - \theta_L) - 3\rho(a^*)\theta_H}{4\theta_H - \theta_L} \tag{44}$$

$$s_L^* = \frac{(2(V - a^*\delta) + \theta_L))(\theta_H - \theta_L) - 2\rho(a^*)\theta_H - \rho(a^*)\theta_L}{(4\theta_H - \theta_L)} \tag{45}$$

Shares become:

$$B_H^* = \frac{2\theta_H + V + \rho(a^*) - a^*\delta}{4\theta_H - \theta_L} \tag{46}$$

$$B_L^* = \frac{2\theta_H}{\theta_L}\frac{\frac{1}{2}\theta_L + V + \rho(a^*) - a^*\delta}{4\theta_H - \theta_L} \tag{47}$$

■

Differently from the multi-channel monopoly case, all the equilibrium values for each channel do not depend just upon the own quality. There is a strategic interdependence between the two broadcasters resulting in prices and shares depending on quality differentiation.

We consider the case of a uniform distribution of advertisers, to get a sharper intuition of our results. Equilibrium solutions of stage 2 rewrites as follows.

Subscription fees:

$$s_H^* = \frac{(V+2\theta_H)(\theta_H-\theta_L) - \frac{1}{4}(1-\delta)(3\theta_H + \delta(5\theta_H - 2\theta_L))}{4\theta_H - \theta_L}$$
$$s_L^* = 2\frac{(V+\frac{1}{2}\theta_L)(\theta_H-\theta_L) - \frac{1}{4}(1-\delta)(\theta_H + \frac{1}{2}\theta_L + 3\delta(\theta_H - \frac{1}{2}\theta_L))}{(4\theta_H - \theta_L)}$$

Viewers' Shares:

$$B_H^* = \frac{2\theta_H + V + (\frac{1-\delta}{2})^2}{4\theta_H - \theta_L}$$
$$B_L^* = \frac{2\theta_H}{\theta_L}\frac{\frac{1}{2}\theta_L + V + (\frac{1-\delta}{2})^2}{4\theta_H - \theta_L}$$

Advertising prices

$$r_H^* = \frac{(\frac{1+\delta}{2})}{(4\theta_H - \theta_L)}\left(V + 2\theta_H + (\frac{1-\delta}{2})^2\right))$$
$$r_L^* = \frac{2(\frac{1+\delta}{2})}{(4\theta_H - \theta_L)}(V + \frac{1}{2}\theta_L + (\frac{1-\delta}{2})^2)$$

4.3 Qualities

We can now solve the initial stage of the game, namely the quality choice.

At stage 1 platforms' profits are respectively

$$\pi_H^* = \frac{(2\theta_H + V + \rho(a^*) - a^*\delta)^2(\theta_H - \theta_L)}{(4\theta_H - \theta_L)^2} - K$$
$$\pi_L^* = \left(\frac{4\theta_H}{\theta_L}\frac{(\frac{1}{2}\theta_L + V + \rho(a^*) - a^*\delta)^2(\theta_H - \theta_L)}{(4\theta_H - \theta_L)^2}\right) - K$$

FOC with respect to quality are respectively:

$$\frac{\partial \pi_H}{\partial \theta_H} = \frac{(2\theta_H+Z)[4(\theta_H-\theta_L)+2\theta_H+Z](4\theta_H-\theta_L)-8(2\theta_H+Z)^2(\theta_H-\theta_L)}{(4\theta_H-\theta_L)^3} = 0 \tag{48}$$

$$\frac{\partial \pi_L}{\partial \theta_L} = 4\theta_H(\frac{(\frac{1}{2}\theta_L+Z)(4\theta_H-\theta_L)(\theta_H-\theta_L)(\frac{1}{2}\theta_L-Z)-\theta_L(\frac{1}{2}\theta_L+Z)^2(2\theta_H+\theta_L)}{(4\theta_H-\theta_L)^3\theta_L^2}) = 0 \tag{49}$$

with $Z = V + \rho(a^*) - a^*\delta$

Conditions (48) and (49) implicitly define the best replies in quality for the two platforms. Unfortunately, the simultaneous solution does not give us a unique outcome. To make our duopoly comparable with the multi-channel case, we decide to focus on an equilibrium with maximal differentiation in quality. Therefore, we restrict the technological range of quality (Θ) to a narrower set $\Theta^D = (\underline{\theta}, \bar{\theta})$ with $\underline{\theta} > \frac{4}{7}\bar{\theta}$. If θ_H and θ_L belongs to this range, we obtain the following result:

Proposition 9 *In the restricted range of qualities $\Theta^D = (\underline{\theta}, \bar{\theta})$ with $\underline{\theta} > \frac{4}{7}\bar{\theta}$ there is a unique local equilibrium of maximal differentiation.*
Equilibrium subscription fees and viewers' shares become:

$$s_H^* = \frac{(V - a^*\delta + 2\bar{\theta})(\bar{\theta} - \underline{\theta}) - 3\rho(a^*)\bar{\theta}}{4\bar{\theta} - \underline{\theta}}$$
$$s_L^* = \frac{(2(V - a^*\delta) + \underline{\theta})(\bar{\theta} - \underline{\theta}) - 2\rho(a^*)\bar{\theta} - \rho(a^*)\underline{\theta}}{(4\bar{\theta} - \underline{\theta})}$$

$$B_H^* = \frac{2\bar{\theta} + V + \rho(a^*) - a^*\delta}{4\bar{\theta} - \underline{\theta}}$$
$$B_L^* = \frac{2\bar{\theta}}{\underline{\theta}} \frac{\frac{1}{2}\underline{\theta} + V + \rho(a^*) - a^*\delta}{4\bar{\theta} - \underline{\theta}}$$

Proof. It is possible to show that if $4\theta_H < 7\theta_L$

$$\frac{\partial \pi_H}{\partial \theta_H} > 0 \tag{50}$$
$$\frac{\partial \pi_L}{\partial \theta_L} < 0 \tag{51}$$

Hence, for every $\theta \in \Theta^D = (\underline{\theta}, \bar{\theta})$ with $\underline{\theta} > \frac{4}{7}\bar{\theta}$, (50) and (51) hold. Therefore

$$\theta_H^* = \bar{\theta}$$
$$\theta_L^* = \underline{\theta}$$

■

For what concerns audience shares, Proposition 9 highlights the effects of our assumption of uncovered market. If the market were uncovered ex-ante, we would have obtained a split in the market in a fixed proportion[18]. Instead, we show that the audience shares depend on the quality distance of the two platforms.

Notice that, in our analysis, the level of qualities are as fixed at maximum differentiation. However, our setting also allows for modelling an endogenous decision on quality levels, though analytically hardly tractable. Nevertheless, our results in the local equilibrium may give a suggestion on how these quality levels would change if the decisions of quality were endogenous.

4.4 Viewers' Surplus

As for the monopoly case, we address the welfare analysis from the point of view of the viewers. Viewers' Surplus in the uncovered duopoly is:

$$SC_D(\underline{\theta},\bar{\theta}) = \int_0^{\beta_{0L}} (u_0)\, d\beta + \int_{\beta_{0L}}^{\beta_{LH}} (u_L)\, d\beta + \int_{\beta_{LH}}^{1} (u_H)\, d\beta \tag{52}$$

At the local equilibrium, we obtain:

$$SC_D^*(\underline{\theta},\bar{\theta}) = \frac{1}{2}\frac{\overline{\theta}}{\underline{\theta}\left(4\overline{\theta}-\underline{\theta}\right)^2}\left(\left(4\overline{\theta}+5\underline{\theta}\right)\left(\overline{\theta}\underline{\theta}+Z^2\right)+2\underline{\theta}\left(8\overline{\theta}+\underline{\theta}\right)Z\right) \tag{53}$$

with $Z = V + \rho(a^*) - a^*\delta$.

5 The Welfare Effects of Competition

Viewers' surplus is an important element to be considered when we analyze the effect of potential competition. In this perspective we first compare our duopoly with the multi-channel monopoly case described in the first section. In this comparison we pay particular attention to the difference between viewers' surpluses and we also consider how prices and audiences change according to the degree of competition.

Proposition 10 *If both the duopoly and the multi-channel monopoly configurations show a situation of maximum differentiation, viewers are better off with more competition (duopoly). That is:*

$$SC_D^*(\underline{\theta},\bar{\theta}) - SC_{MP}^*(\underline{\theta},\bar{\theta}) > 0$$

Proof. Recall equilibrium viewers' surplus in duopoly (ex-ante uncovered) with maximal differentiation (with $\Theta^d = (\underline{\theta},\bar{\theta})$ such that $\underline{\theta} > \frac{4}{7}\overline{\theta}$) from equation (53)

$$SC_D^*(\underline{\theta},\bar{\theta}) = \frac{1}{2}\frac{\overline{\theta}}{\underline{\theta}\left(4\overline{\theta}-\underline{\theta}\right)^2}\left(\left(4\overline{\theta}+5\underline{\theta}\right)\left(\overline{\theta}\underline{\theta}+Z^2\right)+2\underline{\theta}\left(8\overline{\theta}+\underline{\theta}\right)Z\right)$$

[18] See Weeds (2013).

with $Z = V + \rho(a^*) - a^*\delta$, and equilibrium viewers' surplus in the multichannel monopoly from equation (27)

$$SC^*_{MP}(\underline{\theta}, \bar{\theta}) = \frac{1}{8\underline{\theta}}(\underline{\theta}\overline{\theta} + (V + \rho(a^*) - a^*\delta)^2 + 2\underline{\theta}V + \rho(a^*) - a^*\delta)$$

If we compare them we get

$$\begin{aligned} & SC^*_D(\underline{\theta}, \bar{\theta}) - SC^*_{MP}(\underline{\theta}, \bar{\theta}) \\ = & \frac{1}{8\left(\underline{\theta} - 4\overline{\theta}\right)^2}\left((28\overline{\theta} - \underline{\theta})\left(Z^2 + \overline{\theta}\underline{\theta}\right) + 2Z(16\overline{\theta}^2 - \underline{\theta}^2) + 24\overline{\theta}\underline{\theta}Z\right) \end{aligned}$$

with $Z = V + \rho(a^*) - a^*\delta$. The above expression is for sure positive, provided that $\rho(a^*) - a^*\delta > 0$. Notice that this is the case if we consider a uniform distribution of advertisers. Namely, in the uniform case we have $\rho(a^*) - a^*\delta = \left(\frac{1-\delta}{2}\right)^2 > 0$, which gave us:

$$SC^*_D(\underline{\theta}, \bar{\theta}) - SC^*_{MP}(\underline{\theta}, \bar{\theta}) > 0$$

■

As shown in Table 3, this result is driven by lower prices in the duopoly case, provided that $\rho(a^*) - a^*\delta > 0$ (as in the uniform case). In addition, there is a better market coverage by the two competing firms, as emerges from the shares' comparison.[19]

[19] We compare a duopoly of single-channel broadcasters with a multi-channel monopoly broadcaster. We concentrate on a local equilibrium where both market configurations exhibit maximal differentiation in quality. Hence, we must impose some restrictions on the tecnological range Θ, namely $\Theta^d = (\underline{\theta}, \bar{\theta})$ such that $\underline{\theta} > \frac{4}{7}\overline{\theta}$.

Table 3: DUOPOLY vs MULTI-CHANNEL MONOPOLY

Viewers' Surplus	$SC_D^* > SC_{MP}^*$
Channels' Quality Levels	$\theta_H^{D*} = \theta_H^{MP*}$ $\theta_L^{D*} = \theta_L^{MP*}$
Viewers' Fees	$s_H^{*D} < s_H^{*MP}$ $s_H^{*D} < s_H^{*MP}$
Viewers' Market Shares	$B_H^{*D} > B_H^{MP}$ $B_L^{*D} > B_L^{MP}$
Advertisers' Market Shares	$a_i^{D*} = a_i^{MP*}$

Finally we make a last comparison between our duopoly (uncovered) and an ex-ante covered duopoly. If we consider just the restricted range $\Theta^d = (\underline{\theta}, \bar{\theta})$ with $\underline{\theta} > \frac{4}{7}\overline{\theta}$, both configurations show maximal differentiation but different subscription fees and audience shares.

Proposition 11 *Consider the duopoly case. If both the ex-ante covered and the uncovered configuration lead to a situation of maximum differentiation, viewers are better off in the uncovered duopoly.*

Proof. If the market is ex-ante covered we just need one marginal individual β_{LH}. We compute viewers' surplus in the ex-ante covered case using equilibrium values:

$$\beta^*_{LHDcovered} = \frac{1}{3}$$

$$B^*_{HDcovered} = \frac{2}{3}$$
$$B^*_{LDcovered} = \frac{1}{3}$$

$$s^*_{HDcovered} = \frac{2}{3}\left(\overline{\theta} - \underline{\theta}\right) - \rho(a^*)$$
$$s^*_{LDcovered} = \frac{1}{3}\left(\overline{\theta} - \underline{\theta}\right) - \rho(a^*)$$

$$SC^*_{Dcovered}(\underline{\theta},\bar{\theta}) = \frac{-2\bar{\theta}+11\underline{\theta}}{18} + V + \rho(a^*) - a^*\delta \tag{54}$$

We compare this surplus with the one from equation (53), under the constraint $\Theta^d = (\underline{\theta},\bar{\theta})$ with $\underline{\theta} > \frac{4}{7}\bar{\theta}$:

$$SC^*_D(\underline{\theta},\bar{\theta}) - SC^*_{Dcovered}(\underline{\theta},\bar{\theta}) > 0$$

provided that $\rho(a^*) - a^*\delta > 0$ (which is true in the uniform case).

Since we have considered the covered and the uncovered duopoly as two different market configurations, we have to check if this distinction still holds in equilibrium. In equilibrium, the duopoly broadcasting market is ex-post uncovered if: [20]

$$\begin{gathered} \beta_{0L} > 0 \\ \overline{\theta} - \underline{\theta} > \frac{2\overline{\theta}+\underline{\theta}}{\underline{\theta}}(V + \rho(a^*) - a^*\delta) \end{gathered} \tag{55}$$

■

If this were the case, we can provide some more intuitions looking at fees and audience shares. As matter of facts, when quality differentiation is sufficiently high, the covered duopoly shows higher market shares but also higher subscription fees on both channels compared to the uncovered scenario. Higher prices explain the distance between covered and uncovered surpluses. Indeed, the possibility of catching extra viewers, as happens in the uncovered market, disciplines the behavior of platforms in duopoly, making consumers' surplus higher.

It is trivial to show that if:

$$\overline{\theta} - \underline{\theta} > \underbrace{\frac{3}{2}(V + \rho(a^*) - a^*\delta)}_{A} \tag{56}$$

then $B^*_{Hcovered} > B^*_H$ and $s^*_{icovered} > s^*_i$ for $i = H, L$.

Analogously if

$$\overline{\theta} - \underline{\theta} > \underbrace{6\frac{\overline{\theta}}{\underline{\theta}}(V + \rho(a^*) - a^*\delta)}_{B} \tag{57}$$

then $B^*_{Lcovered} > B^*_L$.

Recall from condition (55), that the market is uncovered (ex-post) if

[20] Notice that the market is covered ex-post if condition (55) is not satisfied. However, we neglet this case from our analysis since a comparison between two covered market structures is meaningless.

$$\overline{\theta} - \underline{\theta} > \underbrace{\frac{2\overline{\theta} + \underline{\theta}}{\underline{\theta}}(V + \rho(a^*) - a^*\delta)}_{C}$$

It is possible to show that $A < C < B$. If $\overline{\theta} - \underline{\theta} > B$ then $s^*_{icovered} > s^*_i$ and $B^*_{icovered} > B^*_i$ for $i = H, L$. The covered duopoly has higher prices and larger audiences on both channels. If instead $C < \overline{\theta} - \underline{\theta} < B$ then $s^*_{icovered} > s^*_i$ and $B^*_{Hcovered} > B^*_H$ but $B^*_{Lcovered} < B^*_L$: prices are still higher but now the uncovered has a higher share on the low quality channel. Finally if $\overline{\theta} - \underline{\theta} < C$ the uncovered market becomes covered. However, as already mentioned, a comparison between two covered market structures is meaningless. Given that, when we considered a comparison between an uncovered ($\overline{\theta} - \underline{\theta} > C$) and a covered structure, it must be the case that the covered one privileges the high quality channels and sets higher subscription fees on both channels.

6 Conclusions

In this paper we perform a welfare analysis in a setting of vertical differentiated two-sided broadcasters, where competition prevails on one side of the market, namely on viewers. Broadcasters act as "bottlenecks" between advertisers and viewers by offering sole access to their respective set of viewers. We provide a full characterization of equilibria for what concerns advertising, subscription fees, market shares and qualities, for the monopoly with single-channel platform, multi-channel monopoly and duopoly cases. For what concern the welfare analysis, we focus on the viewers side, calculating the consumers' surplus for each market structure.

Notice that, differently from the ongoing literature on vertically differentiated media, we consider an ex-ante uncovered market. This framework further complicates the model from an analytical point of view, arising multiple equilibria. To overcome this issue we identify a technological range of qualities allowing a local equilibrium of maximal differentiation to exist. Despite this restriction, we strongly believe it is worth to maintain an uncovered set-up since it better points out the effects of competition on audience and prices.

Let us remark that equilibrium qualities depends also on the cost structure used in this model. Under the assumption of fixed costs, monopoly profit function is convex in quality. One might expect that this shape strictly depends on the assumption of K, fixed cost of quality. Indeed, in a single-side framework the standard model of vertical differentiation is solved with a quadratic cost of quality inducing concavity of profit function. However in a two-sided setting the issue of concavity of profit function is more complex. As expected, even in the two-sided approach, linear cost of quality does not solve the problem of convexity of profit function. But, more surprisingly, even increasing marginal cost of quality does not guarantee a well-shaped monopoly profit function. For instance, quadratic cost of quality (see Weeds (2013) do not make concave the monopoly profit function, for what concerns quality, without *ad hoc* assumptions on the derivatives. One possible way out would be having implicit quality cost functions (see Anderson (2007), that, however, should unable us to provide a close solution of the model. Therefore we chose to introduce a simplest cost function and a technological range bounding the levels of quality.

Our results show that the chance of choosing among channels of different qualities is always beneficial for the viewers. In the comparison between the single and the multi-channel monopoly broadcaster, this result is mainly driven by two forces. On the one hand, the possibility of choosing among different qualities; on the other, a larger audience coverage and a pricing effect.

We also prove that competition is beneficial for the audience. The audience surplus is larger in the duopoly configuration then in monopoly setting, when both provide high and low-quality channels. On both channels, subscription fees are lower while the shares of viewer are larger. This result suggests that the ownership in broadcasting markets matters. In this respect regulation should set limits on the ownership of TV channels inducing a more fragmented market structure.

Aside from the issue of ownership, we also prove the beneficial effects of competition by comparing covered and uncovered duopoly. Indeed, the.chance of catching extra viewers disciplines the platforms' behavior in duopoly making consumers' surplus higher. From the policy makers point of view, this result is crucial in the broadcasting sector, where the convergence between television and internet continuously opens up new market segments.

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7 Appendix

7.1 Multi-product Monopoly with Different Costs of Quality (Quality Stage)

Linear Costs [21]

Profits at stage 1 are:

$$\pi_{MP} = \frac{\theta_H}{4} + \frac{(V + \rho(a^*) - \delta a^*)(2\theta_L + V + \rho(a^*) - \delta a^*)}{4\theta_L} - \gamma\theta_H - \gamma\theta_L \tag{58}$$

Looking at first order conditions we get:

[21] Both linear and quadratic costs are assumed to be separable. See Section 2.

$$\frac{\partial \pi_{MP}}{\partial \theta_H} = \frac{1}{4} - \gamma = 0 \tag{59}$$

$$\frac{\partial \pi_{MP}}{\partial \theta_L} = -\frac{1}{4\theta_L^2} (V + \rho(a^*) - \delta a^*)^2 - \gamma < 0 \tag{60}$$

Optimal qualities are:

$$\theta_H^* = \overline{\theta} \text{ if } \gamma < \frac{1}{4} \tag{61}$$

$$\theta_H^* = \underline{\theta} \text{ if } \gamma > \frac{1}{4} \tag{62}$$

$$\theta_L^* = \underline{\theta} \tag{63}$$

The degree of differentiation depends on the cost parameter γ.

- If $\gamma < \frac{1}{4}$ the platform chooses to maximally differentiate the two channels.
- If $\gamma > \frac{1}{4}$ the platform chooses to duplicate the minimum quality

  In the first case profits become:

$$\pi_{MP}^* = \frac{\overline{\theta}}{4} + \frac{(V - \delta a^* + \rho(a^*))(2\underline{\theta} + V - \delta a^* + \rho(a^*))}{4\underline{\theta}} - \gamma(\overline{\theta} + \underline{\theta}) \tag{64}$$

In the uniform case equilibrium values are:

$$a_H^* = a_L^* = a^* = \frac{1-\delta}{2} \tag{65}$$

$$s_H^* = \frac{\overline{\theta} + V - \frac{(1-\delta)(1+3\delta)}{4}}{2} \tag{66}$$

$$s_L^* = \frac{\underline{\theta} + V - \frac{(1-\delta)(1+3\delta)}{4}}{2} \tag{67}$$

$$B_H^* = \frac{1}{2} \tag{68}$$

$$B_L^* = \frac{1}{2} - \frac{\underline{\theta} - V - (\frac{1-\delta}{2})^2}{2\underline{\theta}} \tag{69}$$

$$\pi^*_{MP} = \frac{\overline{\theta}}{4} + \frac{(V + (\frac{1-\delta}{2})^2)(2\underline{\theta} + V + (\frac{1-\delta}{2})^2)}{4\underline{\theta}} - \gamma(\overline{\theta} + \underline{\theta}) \tag{70}$$

Quadratic Costs
Profits at stage 1 are:

$$\pi_{MP} = \frac{\theta_H}{4} + \frac{(V + \rho(a^*) - \delta a^*)(2\theta_L + V + \rho(a^*) - \delta a^*)}{4\theta_L} - \frac{1}{2}\gamma\theta_H^2 - \frac{1}{2}\gamma\theta_L^2 \tag{71}$$

Looking at first order conditions we get

$$\frac{\partial \pi_{MP}}{\partial \theta_H} = \frac{1}{4} - \gamma\theta_H = 0 \tag{72}$$

$$\frac{\partial \pi_{MP}}{\partial \theta_L} = -\frac{1}{4\theta_L^2}\left(V + \rho(a^*) - \delta a^*\right)^2 - \gamma\theta_L < 0 \tag{73}$$

Optimal qualities are:

$$\theta^*_H = \frac{1}{4\gamma} \tag{74}$$

$$\theta^*_L = \underline{\theta} \tag{75}$$

The degree of differentiation depends on the dimension of the technological constraint in respect to the cost parameter γ.

- If $\underline{\theta} < \frac{1}{4\gamma}$ the platform chooses a quality above the minimum.
- If $\underline{\theta} < \frac{1}{4\gamma} < \overline{\theta}$ the platform chooses a quality above the minimum but below the maximum.
- If $\overline{\theta} < \frac{1}{4\gamma}$ the platform chooses to reach the upper bound of the range $\overline{\theta}$.

Hence,with $\underline{\theta} < \overline{\theta} < \frac{1}{4\gamma}$, we get a result of maximal differentiation profits become:

$$\pi^*_{MP} = \frac{\overline{\theta}}{4} + \frac{(V - \delta a^* + \rho(a^*))(2\underline{\theta} + V - \delta a^* + \rho(a^*))}{4\underline{\theta}} - \frac{1}{2}\gamma(\overline{\theta}^2 + \underline{\theta}^2) \tag{76}$$

In the uniform case equilibrium values are:

$$a^*_H = a^*_L = a^* = \frac{1-\delta}{2} \tag{77}$$

$$s_H^* = \frac{\overline{\theta} + V - \frac{(1-\delta)(1+3\delta)}{4}}{2} \tag{78}$$

$$s_L^* = \frac{\underline{\theta} + V - \frac{(1-\delta)(1+3\delta)}{4}}{2} \tag{79}$$

$$B_H^* = \frac{1}{2} \tag{80}$$

$$B_L^* = \frac{1}{2} - \frac{\underline{\theta} - V - (\frac{1-\delta}{2})^2}{2\underline{\theta}} \tag{81}$$

$$\pi_{MP}^* = \frac{\overline{\theta}}{4} + \frac{(V + (\frac{1-\delta}{2})^2)(2\underline{\theta} + V + (\frac{1-\delta}{2})^2)}{4\underline{\theta}} - \frac{1}{2}\gamma(\overline{\theta}^2 + \underline{\theta}^2) \tag{82}$$

7.2 Monopoly (Single-Product)

7.2.1 Monopoly (Single Product): Viewers' and Advertisers' Shares

By considering the individual indifferent between accessing the monopoly platform or not accessing at all, we obtain the demand function by viewers/readers. [22]

$$\beta_{0M} = \frac{s_M + \delta a_M - V}{\theta_M} \tag{83}$$

Since individuals are uniformly distributed between 0 and 1, the demand for the monopoly platform is simply given by the fraction of population with a taste for quality greater than β_{0M} :

$$B_M = (1 - \beta_{0M}) = \left(\frac{V + \theta_M - s_M - \delta a_M}{\theta_M} \right) \tag{84}$$

The amount of advertising for the platform becomes:

$$a_M = 1 - F\left(\frac{r_M}{B_M}\right) \tag{85}$$

[22] This section summarizes the results for a single-channel monopoly case and it builds on the model of Battaggion and Drufuca (2014).

In addition, we present results either for a monopoly choosing the mimimum quality and for a monopoly choosing the maximum quality.

Having defined the demand function of viewers/readers and advertisers, for given prices r_M and s_M, we solve the game backwards, from stage three. Therefore by simultaneously solving the equations (84) and (85) we get:

$$r_M\left(s_M, a_M, \theta_M\right) = F^{-1}(1-a_M)\left(\frac{V+\theta_M - s_M - \delta a_M}{\theta_M}\right) \tag{86}$$

This equation describes how advertising charges react to changes in subscribers' prices, advertising and qualities.

7.2.2 Monopoly (Single Product): Subscription Fee and Advertising Intensity

According to the above assumptions, the platform maximizes profits, subject to a positivity constraint on advertising level.

$$\begin{cases} \max\limits_{a_H, s_H} \Pi_M = B_M(s_M + \rho_M) - K \\ \qquad\qquad s.t. a_M \geq 0 \end{cases} \tag{87}$$

First order conditions with respect to the advertising spaces a_M and subscription fees s_M are respectively:

$$\frac{\partial \Pi_M}{\partial a_M} = \frac{\partial B_M}{\partial a_M} s_M + r_M + a_M \frac{\partial r_M}{\partial a_M} \leq 0 \tag{88}$$

and

$$\frac{\partial \Pi_M}{\partial s_M} = B_M + \frac{\partial B_M}{\partial s_M} s_M + a_M \frac{\partial r_M}{\partial s_M} = 0 \tag{89}$$

Then, according to the literature, we define the advertising revenues per viewer as $\rho(a_i)$

$$\rho(a_i) = \frac{a_i r_i}{B_i} = \frac{a_i F^{-1}(1-a_i) B_i}{B_i} = a_i F^{-1}(1-a_i) \tag{90}$$

We assume $\rho(a_i)$ to be concave in the interval $a \in [0,1]$. Given that $\rho(a_i) = 0$ for $a_i = 0$ and $a_i = 1$, the function is single-peaked.

Using the definition (90) for the monopoly platform we can rewrite optimality conditions, proving the following Proposition.

Proposition 12 *The optimal advertising level of monopoly media platform is:*

$$\rho'(a_M) = \delta$$

Proof. Given (90) for the monopoly platform

$$\rho(a_M) = \frac{a_M r_M}{B_M} = \frac{a_M F^{-1}(1-a_M)B_M}{B_M} = a_M F^{-1}(1-a_M) \tag{91}$$

we have:

$$r_M = \frac{B_M \rho(a_M)}{a_M} \tag{92}$$

Therefore optimality conditions (88) and (89) rewrite into (93) and (94):

$$s_M \frac{\partial B_M}{\partial a_M} + r_M + a_M \left[\frac{\left(B_M \rho(a_M) + \frac{\partial B_M}{\partial a_M} \rho(a_M)\right) a_M - B_M \rho(a_M)}{a_M^2} \right] \leq 0 \tag{93}$$

$$B_M + s_M \frac{\partial B_M}{\partial s_H} + a_M \frac{\partial r_M}{\partial s_M} = 0 \tag{94}$$

By easy calculation, (93) and (94) become respectively:

$$\frac{\partial B_M}{\partial a_M} \left(s_M + \rho(a_M)\right) + B_M \rho(a_M) \leq 0 \tag{95}$$

$$\frac{\partial B_M}{\partial s_H} \left(s_M + \rho(a_M)\right) + B_M = 0 \tag{96}$$

Given that $\dfrac{\partial B_M}{\partial a_M} = -\dfrac{\delta}{\theta_M}$ and $\dfrac{\partial B_M}{\partial s_M} = -\dfrac{1}{\theta_M}$, we get:

$$\frac{\partial B_M}{\partial a_M} = \delta \frac{\partial B_M}{\partial s_M} \tag{97}$$

Therefore, plugging in (95) and (96), we get the following system:

$$\begin{cases} \delta \frac{\partial B_M}{\partial s_M} \left(s_M + \rho(a_M)\right) + B_M \rho'(a_M) \leq 0 \\ \dfrac{\partial B_M}{\partial s_H} \left(s_M + \rho(a_M)\right) + B_M = 0 \end{cases} \tag{98}$$

Finally, if $a_M > 0$ the above inequality is satisfied by equality. Therefore, given that $\rho(a_M)$ is single-peaked, a_M is uniquely determined by the following condition:

$$\rho'(a_M) = \delta.$$

■

We can now solve for the equilibrium values, as stated in the following proposition.

Proposition 13 *With $\rho(a_M)$ concave, we obtain the equilibrium price s_M^* and demand B_M^* as function of qualities, revenues per viewer and advertising level.*

Proof. By plugging the expression for B_M in the optimality condition (96) we obtain:

$$s_M^* = \frac{V + \theta_M - \rho(a_M^*) - \delta a_M^*}{2} \tag{99}$$

Then,

$$B_M^* = \frac{V + \theta_M + \rho(a_M^*) - \delta a_M^*}{2\theta_M} \tag{100}$$

■

The above Proposition 13 shows the result of profit neutrality. In fact, revenues on advertising side are counterbalanced by a decrease on the subscription fees.

7.2.3 Monopoly (Single Product): Platform's quality

In order to solve the quality stage, we maximize the monopoly profit, $\Pi_M\left(s_M^*, a_M^*, r_M^*, \theta_M\right)$, with respect to the quality, θ_M. We obtain the following FOC, subject to $\theta_M \geq 0$:

$$\begin{gathered} \frac{\partial \Pi_M\left(s_M^*, a_M^*, r_M^*, \theta_M\right)}{\partial \theta_M} = \\ \frac{\left(V + \theta_M + \rho(a_M^*) - \delta a_M^*\right)\left(\theta_M - \rho(a_M^*) + \delta a_M^*\right)}{4\theta_M^2} = 0 \end{gathered} \tag{101}$$

Unfortunately, in this general framework we cannot calculate the equilibrium value of θ_M^*.

By calculating the second order conditions we show the convexity of profit function:

$$\frac{\partial^2 \Pi_M}{\partial \theta_M^2} = \frac{\left(\rho\left(a_M^*\right) - \delta a_M^*\right)^2}{2\theta_M^3} \geq 0 \tag{102}$$

Given convexity, the monopoly platform will reach one of the boundaries, choosing $\underline{\theta}$ or $\bar{\theta}$. Hence we describe two possible local equilibria, each of them characterized by a specific configuration of the technological range.

Proposition 14 *In equilibrium, under the technological constraint $\Theta_{RL} = \left[\underline{\theta}, \bar{\theta}\right]$ with $\underline{\theta} = \rho\left(a_M^*\right) - \delta a_M^*$, the monopoly platform chooses the maximum quality. Differently, under the technological constraint $\Theta_{RH} = \left[\underline{\theta}, \bar{\theta}\right]$ with $\underline{\theta} > 0$ and $\bar{\theta} = \rho\left(a_M^*\right) - \delta a_M^*$, the monopoly platform chooses the maximum quality.*

Proof. In the first case we restrict ourselves on the increasing slope of the profit function. By comparing monopoly profit functions in $\underline{\theta}$ and $\bar{\theta}$, respectively:

$$\Pi_M^*\left(\underline{\theta}\right) = \frac{\left(\underline{\theta} + \rho\left(a_M^*\right) - \delta a_M^*\right)^2}{4\underline{\theta}} - K$$

$$\Pi_M^*\left(\bar{\theta}\right) = \frac{\left(\bar{\theta} + \rho\left(a_M^*\right) - \delta a_M^*\right)^2}{4\bar{\theta}} - K$$

we get:

$$\Pi_M^*\left(\bar{\theta}\right) - \Pi_M^*\left(\underline{\theta}\right) > 0$$

For $\theta \in \Theta_{RL}$ profit are convex and increasing in quality. Therefore to maximize profit the monopoly platform sets $\theta_M^* = \bar{\theta}$.

In the second case, we restrict ourselves on the decreasing slope of the profit function. By comparing monopoly profit functions in $\underline{\theta}$ and $\bar{\theta}$, respectively we get:

$$\Pi_M^*\left(\bar{\theta}\right) - \Pi_M^*\left(\underline{\theta}\right) < 0$$

For $\theta \in \Theta_{RH}$ profit are convex and decreasing in quality. Therefore to maximize profit the monopoly platform sets $\theta_M^* = \underline{\theta}$. ■

Considering the uniform case, we can suggest some interesting insights. By easy calculation, in the uniform case with $\rho\left(a_M\right) = a_M\left(1 - a_M\right)$, we obtain:

$$a_M^* = \frac{1-\delta}{2} \tag{103}$$

$$s_M^* = \frac{V + \theta_M - \delta a_M^* - \rho(a_M^*)}{2} = \frac{V + \theta_M - \left(\frac{1-\delta}{2}\right)\left(\frac{1+3\delta}{2}\right)}{2} \tag{104}$$

$$B_M^* = \frac{1}{2\theta_M}\left[V + \theta_M + \left(\frac{1-\delta}{2}\right)\left(\frac{1-\delta}{2}\right)\right] \tag{105}$$

According to the equilibrium solutions of stage 3 and stage 2, the profit function - in the uniform case - becomes:

$$\Pi_M^* = B_M^*(s_M^* + \rho_M^*) - K = \frac{1}{4\theta_M}(V + \theta_M + \left(\frac{1-\delta}{2}\right)^2)^2 - K \tag{106}$$

Given our result on quality if we consider the case of Θ_{RL}, we obtain equilibrium values for subscription fees and viewers' demand:

$$s_M^* = \frac{V + \bar{\theta} - \left(\frac{1-\delta}{2}\right)\left(\frac{1+3\delta}{2}\right)}{2} \tag{107}$$

$$B_M^* = \frac{1}{2\overline{\theta}}\left[V + \bar{\theta} + \left(\frac{1-\delta}{2}\right)\left(\frac{1-\delta}{2}\right)\right] \tag{108}$$

$$\Pi_M^* = \frac{1}{4\overline{\theta}}(V + \bar{\theta} + \left(\frac{1-\delta}{2}\right)^2)^2 - K$$

For the case of technological range Θ_{RH} equilibrium results are unchanged but for quality.

7.2.4 Monopoly (Single Product): Viewers' surplus

Viewers' surplus is:

$$\begin{aligned} SC_M &= \int_0^{\beta_I} (u_0)\, d\beta + \int_{\beta_I}^1 (u_M)\, d\beta \\ &= \frac{1}{2}\left(1-\beta_I^2\right)(\theta_M) + (1-\beta_I)(V - s_M - \delta a_M) \end{aligned}$$

substituting equilibrium values for β_I , s_M , a_M and θ_M, we get

$$SC_M(\overline{\theta}) = \frac{1}{8\overline{\theta}}\left(V + \rho(a^*) - \delta a^* + \overline{\theta}\right)^2 \tag{109}$$

if Θ_{RL} and

$$SC_M(\overline{\theta}) = \frac{1}{8\underline{\theta}}\left(V + \rho(a^*) - \delta a^* + \underline{\theta}\right)^2 \tag{110}$$

if $\Theta_{RH.}$