Forward guidance policy announcements: how effective are they?[1]

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Abstract: We show that forward guidance adds information to the private sector that can enhance effectiveness of policy, in a way and to an extent that depend on the type of information provided by each specific type of announcement. In fact, this can ensure dynamic or even static controllability in a dynamic context.

1. Introduction

In this paper we investigate the foundations of forward guidance (FG) in the light of the theory of economic controllability in a strategic context. Our argument is that the announcement of future policy choices can facilitate the control of the economic system by the policymaker to produce better results, where the term 'control' is used in the sense of the classical theory of economic policy developed by Tinbergen (1952)[2] and others, and recently extended for the case where policies are used in a strategic manner by Acocella, Di Bartolomeo, Hughes Hallett (2013). However, important as that extension might be for creating a realistic representation of how policies are actually used, it does not specify how FG (FG) works, when it is desirable to use it, what announcements should be made, or how to assess the contribution of the different types of FG available to controlling the economy in a helpful way.

In this paper address those issues directly, in more or less that order. The aim is not so much to produce a general theory of FG in policy making[3], but to study the characteristics of the different forms of FG on offer and how they can improve the controllability of the economy. Our approach is to treat FG as a special case of asymmetric information between agents within a sequential series of complete rationally anticipated solutions as the information sets change.

The paper is constructed as follows. The next section illustrates the different notions of controllability we might be interested in. In section 3 we discuss the different forms of FG available. Section 4 then illustrates the information available to the agents in a model with exogenous rational expectations and to players in a strategic game. Section 5 shows how the specific information provided by the different types of FG facilitate controllability and better outcomes, by adding to the information already available to the private sector having rational expectations, or in a two level Stackelberg information game. Section 6 derives

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[1] We have benefitted from discussions with Giovanni Di Bartolomeo. Some parts of section 2 draw on the analysis in Acocella, Di Bartolomeo, Hughes Hallett (2013), Part IV.
[2] An early presentation of the theory, in the dynamic form we are interested in, can be found in Preston, Pagan (1982) and Hughes Hallett, Rees (1983).
[3] For that see Hughes Hallett and Acocella (2016).

explicit policy rules with FG terms in a linear-quadratic context and then measures of its impact on economic performance. Section 7 concludes.

2. Notions of controllability

We begin with a dynamic policy problem where the private sector has rational expectations. We first state the issue of controllability in general terms. Then to illustrate the role of FG, we change some of the assumptions, mainly with regard to uncertainty, the form of game, and the information available to the private sector and show how to calculate the contribution of the FG terms to reaching the policymakers' targets and their importance relative to the other (conventional) parts of the policy problem; the "welfare" contributions of FG in other words.

The economy itself is described by the following difference equation:

(1) $\qquad y_t = A y_{t-1} + B u_t + C y_{t+1|t} + v_t \qquad$ for $t = 1\dots T$, given values for y_0 and y_{T+1},

where $y \in \mathrm{R}^S$ is the vector of the states of the system reflecting the policymaker's target variables, and where $y_{t+1|t} = E\left[y_{t+1} | \Omega_t \right]$ denotes the mathematical expectation of y_{t+1} conditional on Ω_t (the common information set available to all the agents at t) and u_t is the vector of control variables in the hands of the policymaker. Matrices A, B and C are constant, of order S, Sxm, and S, respectively, and have at least some nonzero elements. In this representation, y_0 is a known initial condition; and y_{T+1} is some known, assumed or expected terminal condition which is part of the common information set Ω_t. Finally v_t is a vector of exogenous shocks or other exogenous influences on y_t; it has a known mean but comes from an unspecified probability distribution.

The final form of this model can be written as:

(2)
$$
\begin{pmatrix} y_{1|1} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ y_{T|1} \end{pmatrix}
=
\begin{bmatrix} R_{11} & \cdot & \cdot & \cdot & R_{1T} \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & & \cdot & & \cdot \\ \cdot & & & \cdot & \cdot \\ R_{T1} & \cdot & \cdot & \cdot & R_{TT} \end{bmatrix}
\begin{pmatrix} u_{1|1} \\ \cdot \\ \cdot \\ \cdot \\ u_{T|1} \end{pmatrix}
+
\begin{pmatrix} b_{1|1} \\ \cdot \\ \cdot \\ \cdot \\ b_{T|1} \end{pmatrix}
, \text{ or } \quad y = Ru + b
$$

where $R = T_T^{-1}(I \otimes B)$, $b = T_T^{-1}\left\{ E(v | \Omega_1) + \left(A':0\right)' y_0 + \left(0:C'\right) y_{T+1|1} \right\}$, $\otimes$ denotes a Kronecker product,

and T_T is the Toeplitz matrix in (2), i.e.:

(3) $\qquad T_T = \begin{bmatrix} I & -C & 0 & \cdot & 0 \\ -A & I & & \cdot & \cdot \\ 0 & & \cdot & & 0 \\ \cdot & \cdot & & \cdot & -C \\ 0 & \cdot & 0 & -A & I \end{bmatrix}.$

To represent the available policy choices, we could add the first-order condition of the central bank's loss function derived by minimizing this under the constraint of our model of the economy[4]. In our case, by assuming that the central bank is interested in all the variables of the reduced form system, we could minimize

(4) $\qquad L = \frac{1}{2}E(y_t - \bar{y}_t)'Q_t(y_t - \bar{y}_t) = \frac{1}{2}E\tilde{y}'Q\tilde{y},$

where $\bar{y} \in \mathrm{R}^S$ is a stacked vector of desired target values, Q is a full rank diagonal matrix of assigned weights, and $\tilde{y}$ denotes deviations from desired values. If there are sufficient instruments (m≥S), or sufficient time periods (t≥S/m), it is generally possible to reach those target values, at least in expectation, exactly; otherwise not. This gives rise to the idea of controllability, and ultimately stabilizability.[5]

There are four different notions of controllability in a dynamic setting:

Static controllability: policy instruments have the ability to reach any specified set of target values, from an arbitrary initial position, *in expectation*, within a single decision period.

Multi-period static controllability: policy instruments have the capability to reach any specified sequence of desired target values (from an arbitrary initial position), in expectation, at every point within an interval containing an arbitrary number of consecutive decision periods.

Dynamic controllability: policy instruments have the ability to achieve an arbitrary set of target values, in expectation, at a single point a certain number of periods in the future, given an arbitrary starting point and without concern for what values those targets or other variables might take along the way or after the target values have been achieved.

Path controllability: policy instruments have the ability to achieve an arbitrary set of target values a given number of decision periods after an arbitrary start; and then either hold those values or follow some pre-specified path of target values, again in expectation, at each decision point for an arbitrary period of time thereafter.

Given *S* and *m*, these concepts of controllability define the capacity to reach the desired targets over an interval of length *T*, i.e. [t_0, t_0+T], having started the policy actions in an earlier period t_0–*P*:

Static controllability: $\qquad\qquad\qquad\qquad$ *P* = 0 $\quad$ *T* = 0.

Multi-period static controllability: $\qquad\qquad$ *P* = 0 $\quad$ *T* > 0.

Dynamic controllability: $\qquad\qquad\qquad\quad$ *P* > 0 $\quad$ *T* = 0.

Path controllability: $\qquad\qquad\qquad\qquad$ *P* > 0 $\quad$ *T* > 0.

In this paper, we are specifically interested in static and dynamic controllability. In formal terms, a model or economy is said to be controllable dynamically if a sequence of instrument values $u_1, \ldots, u_t$ can be found that will reach any arbitrary values, $\bar{y}_t$, for the target variables in period *t*, at least in expectation, given an arbitrary starting point y_0. In this case, in contrast to the static controllability case, we are not concerned with the period-by-period controllability of the target variables between periods 1 and $t-1$. Starting from

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[4] As is clear from the literature, policy rules often differ from those derived by optimizing the preferences of the policy makers' constrained by the model of the economy. This is particularly true for the Taylor rule, which can however be interpreted as a simple way of capturing the rule derived from minimizing the central bank's loss function (according to Woodford, 2001: 19, 'the Taylor rule incorporates several features of an optimal monetary policy, from the stand-point of at least one simple class of optimizing models'). Taylor rules can also be used to cross-check optimal policies, because of their robustness to imperfect knowledge of the economy (e.g., Tillmann 2011; Bursian, Roth. 2013).

[5] Hughes Hallett and Acocella (2014).

period 1, dynamic controllability therefore requires a sequence of intended instrument values, $u_{1|1},......,u_{T|1}$ say, that guarantee $\bar{y}_{t|1}$ is reached in period $t=T$. This is possible only if the sequence of policy multipliers and anticipatory effects in the t-th row block of (3), i.e. $R_{t,1} ... R_{t,T}$, is of full rank, i.e. $r[R_{t,1} ... R_{t,T}] = S$, given an arbitrary initial position y_0 and a specified terminal condition $y_{T+1|1}$ (theorem 12.3, Acocella, Di Bartolomeo, Hughes Hallett, 2013).

Types of *FG*

Forward guidance has been introduced as an innovative way of dealing with situations where monetary policy actions have limited effectiveness, especially in proximity of the zero-bound level of interest rates (Woodford, 2013). There are 3 types of FG: 'open-ended'; 'time-contingent' or 'date-based'; and 'conditional' or 'threshold' (CESifo, 2013; Plosser, 2013): Table 1 below. They have typically been used by monetary authorities, but they could also be used by other types of policymakers.

The first type of FG is a loose one. It indicates that the current policy stance will continue for an indefinite period of time. This was the form of FG implemented by the Federal Reserve in August 2003, and again in December 2008, and by the ECB in July 2013. The second type of forward guid-ance was introduced by the Bank of Canada and the Swedish Riksbank in April 2009, and subsequently by the Federal Reserve in August 2011. It announces that the central bank will implement a given (in our case, an accommodative) action for a certain period of time, possibly with an exit clause defined by the evolution of some relevant variables (e.g. inflation being kept at moderate values); Board of Governors of the Federal Reserve System, 2011[6].

Conditional FG – the third type, to which the Federal Reserve shifted in December 2012, and adopted by the Bank of England in August 2013 - indicates the conditions of the economy (in terms of the values of some relevant policy targets) to which the guidance policy is tied. The policymaker can announce its policy target values, the Fed indicated an unemployment rate of 6.5% and an inflation rate of 2.5%, but no policymaker has hitherto announced a precise policy rule for this kind of guidance. A rule of that type needs not only specific target values, but also marginal rates of substitution between targets, as we shall see. Nor has anyone adopted 'history-dependent' rules of the kind advocated by Eggertsson, Woodford (2003). Nevertheless, rules of this kind carry a degree of commitment as well as announcements of future policy.

Forward guidance acts on expectations. Low nominal interest rates expected to last for some period can stimulate investment and possibly consumption. If the current nominal interest rate is near zero, an expected higher inflation rate – e.g. as a consequence of an announcement (or commitment) of the kind suggested by a history-dependent rule - can not only help by keeping expectations of the real rate low, but also by pushing expected real interest rates negative.

Given the vagueness of the period of 'validity' of the promise, the first type of FG is not likely to be particularly effective. Certainly, such announcements have not evinced much reaction in the markets in practice (see, e.g. Kool, Thornton, 2012) However, this does not rule out the possibility that they had some degree of effectiveness since we do not know the impact of the range of possible counterfactuals. Nevertheless the most effective types of FG are likely to be the '*time-contingent*' and '*conditional*' versions.

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[6] Krugman (1999) and Eggerston, Woodford (2003) had suggested to credibly promise a given expansionary stance for even longer than required by the economic situation. This suggestion was followed by the Fed during the 2003-05 crisis, but then blamed it for nourishing the housing bubble, even if the rationale for the promise was the desire to maintain credibility (Gurkaynak, 2012). The initial suggestion has been now replicated by Woodford (2013).

A shortcoming of the former is that it might be interpreted either as a credible promise or as the policy to be expected if policymakers have made the correct predictions on the duration of the change in the level of real variables and low inflation. This ambiguity derives from the fact that time-contingent FG does not indicate the value of targets a given course of policy action is tied to. To be a credible promise this action should also be prosecuted in the case when either exogenous shocks or the endogenous evolution of the system improve the economy's performance in an unpredicted way[7]. But that could imply a suboptimal solution and thus be time inconsistent. Because of this, according to some, time-contingent FG has the nature of a **Delphic prophecy** and is perceived as such by the market (Meyer, 2012).But it can still reduce private sector's uncertainty (Campbell et al 2012). One reason why FG can be effective in these cases is that the fear of time inconsistency may be counterbalanced by the possibility that markets interpret the promise to keep interest rates low even beyond the time when monetary authorities' target values have been reached as an intentional and credible move to create expectations for relatively high inflation and low or negative real interest rates (Eggertsson, Woodford, 2003)[8] This would impart a more powerful incentive to aggregate demand.

Conditional FG has the advantage of being more flexible while indicating values of targets, at least when the current and prospective evolution of the economy is far from satisfying them, that resemble an **Odyssean commitment**. This is likely to change public expectations (Campbell et al 2012)[9]. At the same time, it also implies some parameters of the decision rule being followed by the central bank; that is, of the relationship between the central bank's policy instrument and its targets. The Fed's announcement in 2012 focused on a couple of points (Plosser, 2013), which could be taken as a sufficient condition for successful guidance in this case. If instead of an announcement about the rule, announcements are made about time-contingent or conditional rules, the private sector could 'learn' about the expected policy rule by taking into account and comparing a number of indicators, such as the projections of some relevant variables (interest rates, inflation rates) made by the policymaker, its actual behavior and the evolution of economy.

Conditional FG can be made more effective and a credible promise, if some target variable, inflation say, is made history-dependent by taking account of both the current evolution of the relevant targets and their target values, as suggested by Eggertsson, Woodford (2003) and Woodford (2013). In the case of monetary policy, a history-dependent announcement (or commitment) consists in some rule specifying a policy that should result in higher inflation expectations when the interest rate is bounded at the zero lower bound.

An issue that must be tackled is the distinction between FG and commitment[10]. The former is a kind of announcement (or cheap talk) and differs from the latter as it simply conveys information about intended future policy decisions. The policymaker communicates information concerning its future conduct that, differently from commitment (a binding message), does not tie its hands to a unique course of action, but leaves it some freedom of action under certain contingencies (a contingent rule). That said, one must recognise that there is a continuum between discretion and commitment, as noted by Chari, Kehoe, Prescott (1989) and reiterated by Ridley (2012). Strictly speaking, perfect commitment is only possible for a Ramsey policymaker with complete authority, infinite life, perfect communication and complete credibility (Levin, 2012). To be effective, therefore, FG must have a high degree of credibility, which means that the

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[7] In principle, the opposite case should also be considered, i.e. the economy's situation can deteriorate in such a way as to ask for a stronger accommodative action. However, when the policy stance has already hit the zero interest rate bound, only enactment of other policy actions could deal with the new situation.

[8] However, this does not mean that Eggertsson, Woodford suggest adopting of a purely date-based type of FG, as Woodford (2013) confirms. Their proposal is for a history-dependent rule, as we will shortly see.

[9] Similar kinds of guidance were suggested by Evans (2011, 2012) and Romer (2011).

[10] A distinction commonly which is confused by many central bankers and economists (Blinder, 2009).

policymaker has no incentive to deviate from the announced action except in exceptional circumstances. This gives an incentive to the private sector to behave in the desired way; this is implicit in the complete model solutions we used above at (3). Then both parties would prefer this course of action to others. Perfect commitment to some course of action would guarantee those actions will be chosen in any case, whatever external shocks may hit the economy, even if this is short sub-optimal for the policymaker in the short term. But if the situation and the foreseeable future are such that the circumstances appear to be as exceptional, FG can still be effective even if it does not imply perfect commitment. For that reason, the announcements might not cover all aspects of the future policies, only some of them. Information available with rational expectations and in strategic games

In this section we discuss the kind of information that is supposed to be available to agents with rational expectations in a strategic policy game. The case where there is a single policymaker and the private sector is assumed to have rational expectations is equivalent to a Stackelberg game with the private sector as the leader and the policymaker as the follower (see Acocella, Di Bartolomeo, Hughes Hallett, 2013, sect. 4.5). Each player is assumed to know either the model of the economy and the strategy chosen by his opponent in the case of a Stackelberg follower; or the model of the economy and his opponent's reaction function in the case of a Stackelberg leader or for players playing Nash. To be sure, that reaction function is different from the rule adopted by the central bank in order to respond to the evolution of the economy; that is, from the rule it uses to set its policies in response to gaps between the current values of the target variables and their target values. However, this rule is equivalent to the leader's quasi-reaction function, or best response function defined in terms of targets rather than instruments.[11]

For simplicity, consider the case when the private sector has rational expectations. In this case the private sector knows the economic model (1), including any policy rules, and exploits all the information available at time t_0 and correctly forecasts the expected value of the relevant variables of the model. If the problem is a Stackelberg game, it is assumed that the private sector knows the economic model and the reaction function of the central bank. The latter is equivalent to a quasi-reaction function obtained by minimizing the central bank's loss function under the constraint of the model. Hence the private sector knows both the target values and the marginal rates of substitution among them. In addition, both the private sector and the policymaker share the same model of the economy. Both agents then share the same information set.

Under the assumptions of *perfect certainty*, symmetric information about the model of the economy (including the policymaker's current and future payoffs) and rational expectations, FG can add no extra information to that available to private agents. With these (unrealistic, but probably useful) assumptions, certainty equivalence, $v_t = 0$, holds. Even if the policymaker has more than one target but only one instrument, it should have controllability of the economy in a dynamic setting. More generally, when $T \geq S$ and $t < T$, but the horizon is long enough to fulfill the rank condition $r[R_{t,1}....R_{t,S}] = S$, dynamic controllability is guaranteed even without forward looking expectations. Rational expectations accelerate the possibility for the policymaker to control the economy from any date, as the private sector anticipates the future behavior of the policymaker and knows that the latter can control the economy.

We now depart from the assumption of certainty equivalence, retaining the other assumptions. We can start by assuming that there are additive shocks to the economy, $v_t \neq 0$ in (1). This makes future reactions of the policymaker uncertain simply because shocks to the economy will occur. But the assumption of

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[11] For a formal definition of the quasi-reaction function: see Acocella, Di Bartolomeo, Hughes Hallett (2013).

rational expectations, and knowledge of the policymaker's preferences, will allow us to deduce the necessary reaction function. Again, no announcement could be informative in these conditions. In order for an announcement to add information to that already available to the private sector, one must add uncertainty of ignorance about the policymaker's future decisions. Announcements can make public action (more) effective either as they can add information on the future decisions to be expected or about the model or loss function itself. While open-ended and time-contingent FG are announcements which add information about actions; conditional FG supplies information about the preference function as well.

Information added by FG with RE and its role in facilitating controllability

4.1. The Coverage of Uncertainty

In order to see when and how FG can add to the information already available to a private sector with rational forward looking expectations, we must move outside the model of perfect certainty or additive uncertainty. Depending on the case, we may or may not retain the other assumptions above.

We show that the nature of information, and our degree of confidence in it, is different for the different types of FG due to the different forms of uncertainty covered by different announcements. As a consequence, the impact on controllability differs. In reality, uncertainty is widespread. It can refer to the model's correspondence to reality, or to the possible occurrence of shocks. In addition, it can involve the policymaker's preference function[12] (and in this case it is, at least as a first approximation, asymmetric) and, as a consequence, change the policy rule or the expected future instrument values. As a result, we find that different types of FG are promises with different degrees of credibility and a different 'coverage' of uncertainty.

4.2. Additional information provided by open-ended FG.

We will assume, for now, that the model is additively uncertain, the preferences of the policy makers are unknown. Without loss of generality, we further assume that $m = 1 < S$ for our policymaker and that $T \geq S$.

In general, FG implies an addition of announced (intended or expected) values for policy instruments to be chosen in any period $t > 1$. In fact, by open-ended FG we mean the policy maker announces that the current policy stance will last for an indefinite period of time. But there is no **certain** promise for future policies, $t > 1$ onwards. This implies that elements of $(u'_{2|1}, \ldots, u'_{T|1})'$ in (3) have different and perhaps subjective degrees of likelihood. The fact that continuation of the current policy is uncertain – and decreasingly probable with the elapse of time – makes it clear why this kind of FG can have a beneficial role, but may not be very effective.

That said, different types of uncertainty (or ignorance) about the policymaker's preferences may reduce or widen the benefits from making announcements of future policy. Ignorance of the nature and of the precise number of targets and target values pursued by the policymaker and the marginal rates of substitution between the targets will increase ineffectiveness. By contrast, if the private sector knew even roughly the number of targets (as a result of general information about the policymaker's 'mission', or as an implication of other announcements), we could say that ineffectiveness will be lower, the lower is the number of targets S pursued by the policymaker and the higher is the number of policy instruments. Hence

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[12] In a dynamic setting, ignorance or uncertainty about the policymaker's future preferences can also affect the policymaker themselves: for example, if the composition of the decision making committees may change in the future. This would be a very difficult case to deal with in analytical terms and we will rule it out in this study.

a high targets/instruments ratio will typically lower the guidance's credibility. To some extent the credibility of the announcement will also depend on the reputation of the policymaker and the distance between the current and the intended/desired future states of the economy.

This kind of FG can give no long term assurance not only about shocks; nor about changes in policy or even in the target values deemed necessary.

4.3. The additional information provided by time-contingent FG

As noted in section 2, time contingent FG avoids uncertainty as to the timing of an exit. If the private sector knows the number of targets (as assumed in the previous sub-section), maintenance of the current policy is guaranteed with certainty, or reasonable certainty, for a certain length of time which enhances policy effectiveness. In this case, assuming dynamic controllability, we can say that clarifying the horizon of current policy can help to bridge the gap between the number of instruments and the number of targets. Obviously, the possibility to control the system depends on both the duration of the policy (let us call it t^*) and the planning/forecasting horizon of the private sector. If the latter is longer than the former, the effectiveness of the policy is weak and dynamic controllability of the system is not guaranteed. Effectiveness is also lower the higher is the number of targets pursued by policymakers and the lower is the number of policy instruments.

Let us assume that time-contingent guidance offers information for 2 periods beyond the current period. This implies $t^*= 3$. If the policymaker has a reputation for sticking to its announcements, information about the policy stance at time $t = 3$ is as reliable as that for time $t = 2$, etc. More generally, if $t^*= T$, the elements of $(u'_{2|1}, …, u'_{T|1})'$ in (3) will all be known (or accepted) with reasonable certainty, which makes the system dynamically controllable from $t=1$, given the rank condition $\mathrm{r}[R_{t,1}....R_{t,S}] = S$ is satisfied at $t=1$, even if the policymaker has only one instrument and the number of its targets is $S = T$.

A problem could arise from the fact that, due to various shocks, this type of FG may become suboptimal after the period in which the targets are hit. Although no promises were made for after t^*, this could give the policymaker an incentive not to continue the announced policies after reaching their targets, creating an illusion of time inconsistency and reducing the credibility of the original guidance. If these un-expected shocks are small, at least for some periods, the risk of sub-optimality and change in the policies is low. But even in the case where this risk is not negligible, the policy announcement can still be credible and effective for two reasons.

The first has to do with the fact that the private sector can trust the announced policy: even if this were suboptimal, private agents may in fact accept that the possible cost to the policymaker of sticking to the announced policy is a necessary cost to maintain commitment. This will make the announcements credible.

Second, commitment apart, because the economy is controllable in the same manner from the first period after the guidance either starts or expires, given dynamic controllability, the new policies will be optimal in the same way - no promises having been given about the old policies beyond t^*. The credibility of the new guidance is not impaired (unless $T < t^*$).

There is a third possibility, where the announced policy remains optimal even if the horizon to which the announcements refer is beyond the date at which policymakers expect their targets to be reached. This is the case where the necessary credibility is attached to the policy rule, rather than specific policy values; for example, FG generated by a history-dependent rule of the kind suggested by Eggertsson, Woodford (2003).

This type of guidance differs from a time-contingent ('fixed-date') announcement. It is in fact a promise of future policies to be calculated contingent on the, as yet unknown, future conditions and the distance between those conditions and current target values. In this type of guidance, policy values are even not mentioned by the policymaker – only the contingent rule by which they will be derived. Even if this type of guidance is different from traditional announcements, the fact that the specific policies are known to follow a well defined and trusted rule [for example, by keeping expectations of inflation high beyond its target value in a severe recession] may influence the private sector and add to the effectiveness of time-contingent guidance. Such policy rules are not necessarily optimal however, and if not their credibility could erode over time.

4.4. The additional information provided by conditional FG

In the previous two sub-sections we assumed that the economy's outcomes are uncertain because of additive shocks, while the policymaker's payoff is unknown. An understanding of at least some parameters of the policy rule followed by the central bank or policymaker – rather than the future decisions themselves – adds knowledge useful in the sense that it helps control the economy. In July 2013, Fed chairman Bernanke, in confirming the December 2012 decision of the FMOC, made it clear before the Congress that, first, no exit policy would be adopted 'at least as long as the unemployment rate remains above 6.5 per cent and inflation and inflation expectations remain well behaved in the sense described in the FOMC's statement' (implying long-term inflation below 2 per cent). In addition, 'the specific numbers for unemployment and inflation in the guidance are thresholds, not triggers. Reaching one of the thresholds would not automatically result in an increase in the Federal funds rate target' (Bernanke, 2013).

In this case the Fed not only indicated, in the first part of the statement, the target values of the central bank's loss function, but also made a step towards the kind of guidance (or commitment) suggested by Krugman (1998) and Eggertsson, Woodford (2003). Comparing the Fed's target values with reality (recall that the US unemployment rate had stagnated above 8% for the whole year 2012, with hardly a reduction, which was well above the Fed's target value) could be used to justify policies to boost investment. Similarly, average monthly inflation rate since May 2012 had been 1.7-1.8%, comfortably below 2%, but with highs of 2.0-2.2% in a couple of months. The Fed was thus delivering a clear message: the current stance would be likely to continue also after reaching one of the target values. In particular, hitting the inflation target value would not necessarily imply a halt to the expansionary policy, as long as the other target had not been hit. The whole announcement is broadly in line with Eggertsson and Woodford's proposal. For our purposes[13] we can interpret this as due to the fact that, even if Bernanke's announcement referred to target values and did not include the marginal rates of substitution of Fed's loss function, there was a clear indication of substitutability between the two targets. All in all, instead of announcing continuation of the current policy for a given period – as in the time-contingent case – the policymaker promised to continue current policies for the period necessary to hit both targets, or at least a satisfactory mix between them. This kind of announcement is likely to be informative for the private sector.

The central bank could give a more complete form of conditional FG, by indicating the Taylor rule it follows. In this case it would also announce the rates of substitution among its targets. If we stick to a rational expectation framework, this information should be enough to deduce the same implications as Eggertsson, Woodford's (2003). Communicating the Taylor rule would in practice be a kind of (soft) commitment, the

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[13] We are reasoning not in terms of commitment, as Eggertsson, Woodford (2003) appear to do, but of announcements as purely adding information to the system. Obviously, however, this involves some credibility argument. But we have already argued that the difference between foward guidance and commitment may become very thin.

possibility still being open of a change in the parameters of the Taylor rule (as well in the nature of targets) favored by monetary authorities, following the emergence of new unexpected shocks or a change in the orientation of those charged with delivering monetary policy.

In formal terms, conditional FG adds to private information sets, information of the follow-ing kind to that already available from knowledge of the model:

(5) $\qquad u_t = g\ (y_t, \bar{y}_t)$,

The details of this relation may however be different in different cases. For example, were the policymaker a central bank, *complete* conditional FG would take the form of announcing its chosen Taylor rule. Equation (6) would the take the form:

(6) $\qquad i_t = \pi_t + r_t^* + a_\pi(\pi_t - \pi_t^*) + a_y(y_t - \bar{y}_t)$

where i_t is the bank's policy rate (short-term nominal interest rate, or the Federal funds rate in the US), π_t is the rate of inflation, π^* is the target rate of inflation, r_t^* is the assumed equilibrium real interest rate, and y_t is the logarithm of real GDP, and $\bar{y}$ that of potential (or target) output.

Announcing such a rule would offer information about the bank's targets, its target values and the marginal rate of substitution between inflation and income (or unemployment). In the case of the Federal Reserve policy in December 2012, only π^* and $\bar{y}$ were announced, not the rate of substitution between output (or unemployment) and inflation.

4.5. Some considerations common to the different types of FG

First, the pros and cons of the various types of FG for enhancing controllability:

Table 1: The *pros* and *cons* of FG for system controllability

Forms of guidance	Pros	Cons
Any type of guidance in the basic model with certainty (sect. 4.1. for other assumptions)	None	None
Open-ended guidance	1. Higher effect in the short run, with high reputation, high *m*, low *S*, low expected changes of the economy	1. low effect in the long run, with low reputation, etc. 2. private sector ignores targets, target values and mrs
Time-contingent FG	1. Higher effect for high *t**, high reputation and high *m*, low *S*, low time horizon of the private sector, low expected changes of the economy 2. high credibility if announcement is interpreted as a commitment 3. lasting policy as a way to stimulate expansion at the Zero Lower Bound	1. Possibility that the announced policy is sub-optimal at some future *t*

Conditional FG	1.Covers (partial or total) ignorance of the loss function	1. Partial assurance in case of shocks to the economy 2. No info (until now) about the MRS 3. No coverage for changes in the arguments and parameters of *L* or the policy rule

We must now clarify some methodological aspects of our analysis. In section 4.1, we saw that FG can have no effect under the strict assumptions of the basic model. This implies that, in order to explain its effectiveness, some additional degree of realism must be added to the model. To that end, we have substituted additive uncertainty in the model, and ignorance about the loss function, with knowledge in the form of announcements of future values for the policy instruments or of parts of the loss function. For conditional FG, partial knowledge of the loss function is introduced but marginal rates of substitution are left unknown. The form of conditional guidance used by the Fed in 2012, in fact removed only uncertainty about target values in the loss function. Announcing the policy rule would add information as to the marginal rates of substitution, leaving only uncertainty as to future changes of the loss function.

5. How to measure the importance and potential impact of Forward Guidance

In order to measure the impact of, and information content in, FG we need to define a point of comparison. There are three possibilities:

FG vs no FG, where no FG should be taken to mean no previous FG. Since the behavioural model will contain forward looking expectations (otherwise FG has no meaning or is constrained to have no effect), this means a comparison with the case where there are autonomous or model consistent expectations held by the private sector, with no previous policy indications from the central bank or fiscal authorities.
A change in FG from previous FG announcements.
FG vs the case the expectations for policy that the private sector would have had in the absence of any announcements.

The first case is rather unlikely since the policymakers always influence the private sector's expectations in practice, at least in terms of outcomes if not policy settings. However, whether they induce expectations for future policy values at the same time (case 3) is moot since, multiperiod static controllability apart, that would in general depend on the private sector knowing the authorities' policy rule as well.[14] Thus, de facto, we will almost always be concerned with case 2, the conventional case where central banks and governments publish forward interest rates, expected budget figures, etc.

Appendix A to this paper shows how to calculate the relative success in welfare or objective function terms of a deterministic policy rule; that is, open loop policies given the information set available, say Ω_1, at that point. This would be the common information set available to all. It also shows how to calculate the relative importance of the stochastic component of the policy rule for the same problem; that is, the amount by which the combined policy rule can be expected to smooth any stochastic shocks hitting the economy (a feedback element) plus the amount by which FG can improve the outcomes in welfare terms by shifting expectations – that is the contribution made by using the private information in the policymakers'

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[14] Cases 1 and 3 correspond to Delphic and Odyssean FG respectively. Where the conditions for static controllability do not apply in each period, policy values to achieve the private sector's expected outcomes are not well (uniquely) defined - being dependent on dynamic controllability (Hughes Hallett et al, 2012).

information set (the closed loop elements in an overall policy rule).[15] These two compon-ents are given by L_d^* and L_s^* in equations (19) and (20) respectively of Appendix A. It only remains to show how the second, based on the policymakers' own information, can be constructed from the various forms of FG outlined in section 4. The comparison of L_s^* to L_d^* will show how important FG is to the policy rule itself.

Either or both of these two measures can also be compared to the importance of success of the stochastic outcomes from an open loop, first period certainty equivalent strategy; the counterpart to L_s^* when there is no attempt to control the impacts of any random shocks $v^* = \frac{1}{2}trQ\Sigma_1$ given below equation (20). Presumably, if we are serious about measuring the contribution of FG in any particular case, the comparison between L_s^* and v^* relative to L_d^* and/or L_s^* will be the main focus.

How can the contribution of FG, as distinct from information already available in a common information set, be introduced into the performance measure L_s^*? There are a number of possible components to this:

1) Where the policymakers have private information or projections of the errors in the term b in (3) that differ from the common information, or more likely from the default position of certainty equivalence. If this FG is accepted by the private sector we subtract $\Delta b = T_T^{-1}\{v_p - v_g\}$ from the variable c throughout the decision rules, and hence L_d^*, in Appendix A[16]. That change would also affect σ, and *hence L_s^** which becomes smaller to the extent of $\frac{1}{2}trQ\{V_p^*(\sigma_p) - V_g^*(\sigma_g)\}$. If, on the other hand, the private sector disregards the FG offered completely, we only have to change the parts involving the stochastic term σ and hence L_s^*. Note $v^* = \frac{1}{2}trQ\Sigma_1$, the value of L_s^* when the authorities make no attempt to control the impact of external shocks, is not affected in either case.

2) In the same way, where private information differs from FG figures for the announced or expected forward policy variables, we **add** $\Delta b = T_T^{-1}(B\otimes I)\{u_p - u_g\}$ to c or σ in (9) before calculating L_d^* and L_s^* as above, where u_g contains the FG elements and u_p what the private sector might have expected before the announcement. This calculation would be what most people think of as the gains, or otherwise, from FG and therefore the point of most interest.

3) If the differences involve differences in understanding how the economy works, and FG if offered on that point, we subtract $\Delta b = \{R_p - R_g\}u_g$ from c and/or σ before calculating L_d^* and L_s^*.

4) If new target values are offered as part of the FG plan[17], we need to subtract $\Delta b = -\{\overline{y}_p - \overline{y}_g\}$ from c and/or σ before calculating L_d^* and L_s^*.

Thus the information value of FG can be evaluated in terms of the improvements or gains in the policy outcomes, $\Delta L_d^*/(L_d^* + L_s^*)$, or in terms of the improved stability/damping of shocks $\Delta L_s^*/(L_d^* + L_s^*)$. Likewise, the question of whether FG has contributed more to an improvement in policy out-comes, or in enhanced stability and lower uncertainty, can be evaluated by looking at $\Delta L_s^*/\Delta L_d^*$ relative to the initial ratio L_s^*/L_d^*.

Commentary: Two points. These calculations can easily be done for all the main forms of FG: open ended/unconditional, time contingent, and conditional. The first form just requires a sufficiently large value

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[15] For the formal distinctions between open loop, feedback and closed loop control – see Acocella, Di Bartolomeo, Hughes Hallett (2013, chapter 3).

[16] where v_p represents private sector information or the certainty equivalent figure; and v_g the FG figure.

[17] For example, where graduated reductions in debt or deficit ratios (Greece), or inflation rates (Argentina) are foreseen; or where a revised recovery plan is introduced (France, Italy).

of T; the second a specific or predetermined value of T; the third, a value of T calculated by simulating the model (1) forward from $t=1$ till the conditioning variable (unemployment, a sustained period of growth, or inflation perhaps) has passed a certain specified threshold value in y_{T-1}.

Second, it is important to emphasise that they represent an ex-ante evaluation of the information value of FG in general. We must be able to judge the usefulness of alternative policy regimes before we become committed to them. Post mortem assessments can only judge how successfully that regime performed in particular cases. They cannot distinguish a well designed policy from one that has just been "lucky".

A number of examples can be presented to strengthen our argument. For simplicity's sake, we only refer to Hughes Hallett, Acocella (2016, sect. 5) and Hughes Hallett, Acocella, Di Bartolomeo (2012). Some of them are reproduced in Appendix B

6. Conclusions

We have shown that FG adds information to the private sector that can enhance effectiveness of policy, in away and to an extent that depends on the type of information provided by each specific type of announcement. In fact, this can ensure dynamic or even static controllability in a dynamic context.

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APPENDIX A: Derivation of the value of FG policy announcements[18]

In our framework, optimal policies based on any given information set (Ω_1 say) are obtained by minimizing (4) subject to the constraints given by the model of the economy (2). Converting the latter to a matching "deviations from desired targets" format,

(7) $\qquad\qquad \tilde{y} = Ru + c$ where $c = b - \bar{y}$,

we get the certainty equivalent optimal decision rule:

(8) $\qquad\qquad u^* = -(R'QR)^{-1}R'Qc.$

However if the optimization is done *without* invoking certainty equivalence, we have the stochastic form of the same rule: that is (8) with c replaced by $b + s - \bar{y}$, where s is the additive uncertainty component of the model: $s = T_T^{-1}\{v - E(v|\Omega_1)\}$.

An alternative solution to the problem allows us to derive both forms of policy rule together and thereby split the success ("welfare" value) of the policies between the underlying certainty equivalent rule, and that due to FG under different information sets. We can do that as follows. We split the additive errors term in the model into deterministic and stochastic components: $E_1(b) + \eta$, where $E(\eta|\Omega_1) = 0$. The model, as seen by the policy makers, then decomposes into deterministic and stochastic components:

(9) $\qquad E_1(\tilde{y}) = RE_1(u) + E_1(c) \qquad$ and $\qquad \eta = R\xi + \sigma$

where $\eta = y - E_1(y), \xi = u - E_1(u)$ and $\sigma = c - E_1(c)$, and $E_1(\cdot) = E(\cdot|\Omega_1)$ is a convenient shorthand. The objective function likewise decomposes into its deterministic and stochastic parts:

(10) $\qquad L = \frac{1}{2}E_1(\tilde{y})'QE_1(\tilde{y}) + \frac{1}{2}E_1(\eta'Q\eta)$

The constrained objective function to be minimized is then,

(11) $\qquad L = \frac{1}{2}[RE_1(u) + E_1(c)]'Q[RE_1(u) + E_1(c)] + \frac{1}{2}E_1[R\xi + \sigma]'Q[R\xi + \sigma].$

Thus, our policy problem involves two disjoint positive definite quadratic forms in separate choice variables

(12) $\qquad min_u L(\tilde{y},u) = min_{E_1(u)}\{L_d[E_1(u), E_1(c)]\} + min_\xi\{L_s[\xi,\sigma]\}$

where $L_d[\cdot,\cdot]$ contains the first term in (11), and $L_s[\cdot,\cdot]$ the second. The overall optimal policy choice is then the sum of the optimizer of the deterministic part of the problem $L_d[\cdot,\cdot]$, and of the stochastic part $L_s[\cdot,\cdot]$. The solution to the first part of the problem, as already foreshadowed at (8),

(13) $\qquad\qquad E_1(u^*) = -(R'QR)^{-1}R'QE_1(c).$

This is the deterministic control, and represents an open loop strategy conditional on the initial information set available in period 1.

The second part of the problem, without invoking certainty equivalence, involves minimizing

(14) $\qquad\qquad \frac{1}{2}E_1(\eta'Q\eta) = \frac{1}{2}\{tr\ QV_1(\eta)\}$

subject to the right hand equation in (9), where $V_1(\eta)$ is the conditional variance of $\tilde{y}$, by choice of ξ. Obviously we cannot pick an actual value for ξ since it will be a function of σ, which is not a variable under

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our control. Indeed, ξ is purely random and therefore unknown a priori. But we can pick the transformation that determines η, given σ, to minimize the conditional variance of η: minimal stochastic disturbances to $\tilde{y}$. That is to say, we pick the transformation of σ in such a way that, whatever the shocks, the disturbances in the targets are minimized in the terms of $E_1(\eta'Q\eta)$.

Let that transformation be of the form $\xi = G\sigma$, so that $\eta = (RG + I)\sigma$. Our minimization problem becomes

(15) $\qquad min_G\{L = \tfrac{1}{2}trQV_1(\eta) - tr\Lambda[V_1(\eta) - (RG + I)\Sigma_1(RG + I)']\}$

where $\Sigma_1 = E_1(\sigma\sigma')$, and Λ is a symmetric (S+m)T matrix of Lagrangian multipliers. The necessary first order conditions are:

(16) $\qquad \partial L/\partial G = R'\Lambda(RG + I)\Sigma_1 = 0$

yielding $G^* = -(R'\Lambda R)^{-1}R'\Lambda\Sigma_1$ as the optimal choice. But $\partial L/\partial V_1(\eta) = 0$ implies $\tfrac{1}{2}Q = \Lambda$ at the optimum. Lastly, $\partial L/\partial\Lambda = 0$ shows the targets to have a minimized variance of

(17) $\qquad V_1^*(\eta) = [I - R(R'QR)^{-1}R'Q]\Sigma_1[I - R(R'QR)^{-1}R'Q]'.$

Combining these results, we may write the optimal policy choices as

(18) $\qquad u^* = -(R'QR)^{-1}R'Q[E_1(c) + \sigma] = u_d^* + u_s^*$

where u_d^* is the deterministic open loop policy term at period 1, and u_s^* the stochastic innovations term.

We are now in a position to make numerical comparisons of the deterministic and stochastic parts of the objective in (12): $L_d[\cdot,\cdot]$ and $L_s[\cdot,\cdot]$. Using (17) and (13) in (11), the optimized objective (or "welfare") function decomposes into its deterministic component

(19) $\qquad L_d^* = \tfrac{1}{2}[E_1(c)]'[Q - QR(R'QR)^{-1}R'Q][E_1(c)]$

and its stochastic component

(20) $\qquad L_s^* = \tfrac{1}{2}trQV_1^*(\eta).$

Either of these terms may be compared to the stochastic outcome from an open loop, first period certainty equivalent strategy; the counterpart to L_s^* when there is no attempt to control the impacts of any random shocks $v^* = \tfrac{1}{2}trQ\Sigma_1$.

Appendix B: The value of FG

For illustration, consider a single equation RE model, in the form of (1), with dynamics:

(28) $\quad y_t = ay_{t-1} + by_{t+1}^e + cu_t + \varepsilon_t$

Such a model can be derived from a conventional New Keynesian model of the type used by Mishkin (2002), to assess the effectiveness of Taylor rules for controlling inflation and stabilizing the output gap. That is, we can start from:

(29) $\quad \pi_t = (1-\lambda)\beta E_t \pi_{t+1} + \lambda \pi_{t-1} + \kappa z_t + \phi f_t + v_t$

(30) $\quad z_t = \varsigma E_{t-1} z_t - \sigma(i_t - E_t \pi_{t+1}) + \chi f_t + \eta_t$

where (29) is an aggregate supply equation with dynamics, π_t is the rate of inflation, z_t the output gap; f_t the stance of fiscal policy; i_t the interest rate (a monetary policy instrument); and v_t and η_t are random shocks.[19] Equation (30) is a forward looking IS curve.

Eliminating z_t between (29) and (30) yields:

(31) $\quad \pi_t = (1-\lambda)\beta E_t \pi_{t+1} + \lambda \pi_{t-1} - \kappa\sigma(i_t - E_t \pi_{t+1}) + \varepsilon_t$

where $\varepsilon_t = \kappa E_{t-1} z_t + (\kappa\chi + \phi) f_t + \kappa\eta_t + v_t$ represents a composite of "shocks" exogenous to both monetary policy and the economy. This definition of ε_t involves a simplification, in that $E_{t-1} z_t$ should be an endogenous (rational) expectation of the output gap. Classifying it as part of the composite error term is to recognize that in reality the private sector will either have extraneous expectations for the output gap because it cannot measure accurate rational expectations for the output gap; or because the private sector is unable to separate cyclical from structural changes in potential output in real time. Agents therefore are likely to use simple extrapolations for $E_{t-1} z_t$ instead. Given that, (31) is a special case of (28) with $b = (1-\lambda)\beta + \kappa\sigma$; $a = \lambda$; $c = -\kappa\sigma$.

a) Forward Guidance in Rule Based Monetary Policies

Stabilization can now be studied by introducing a policy rule to control (28) over the near future. That requires us to control the expected target outcome (i.e. FG) as well:

(32) $\quad u_t = k y_{t-1} + d y_{t+1}^e + k_2 \quad$ say.

Substituting (32) into (28) to show the behavior of the economy under control, we get:

(33) $\quad y_t = (a + ck) y_{t-1} + (b + cd) y_{t+1}^e + \varepsilon_t^*$

where $\varepsilon_t^* = \varepsilon_t + k_2$. The evolution of this economy under control is now defined by renormalizing (33) on its lead term, taking expectations conditional on the information available in period t, dropping superscript "e" for simplicity: $y_{t+1} = (b + cd)^{-1}[y_t + (a + ck_1)y_{t-1} + \varepsilon_t^*]$.

But the impact of policy, and, within that, the information value of FG, can be obtained from (19)-(20). To evaluate those expressions, we have to reformulate (33) to obtain its final form (2) for a two-period control problem (at least) since we have to control the one-period ahead expectation of the target in order to manage the current target value. Thus,

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[19] Many New Keynesian models specify marginal costs, mc_t, as the push factor in inflation, in place of f_t. In this case **Errore. L'origine riferimento non è stata trovata.** would have $\kappa\chi f_t + \phi mc_t$ in ε_t in place of the $(\kappa\chi + \phi) f_t$. This alternative makes no difference to the problem as long as mc_t remains exogenous (at least in the short term).

(34) $\quad \begin{bmatrix} y_t^e \\ y_{t+1}^e \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} 1 & (b+cd) \\ (a+ck) & 1 \end{bmatrix} \left\{ \begin{bmatrix} \varepsilon_t^{*e} \\ \varepsilon_{t+1}^{*e} \end{bmatrix} + \begin{pmatrix} (a+ck)y_{t-1} \\ (b+cd)y_{t+2} \end{pmatrix} \right\}$

where $\Delta = 1 - (b+cd)(a+ck)$; and y_{t-1}, y_{t+2} are known initial, assumed terminal conditions respectively. This version of the model is now in the form of (2) if we pick out the remaining policy choices (k_2, possibly f_t) from ε_t^{*e} and ε_{t+1}^{*e} and allocate the rest to vector b in (2). For example, in the notation of (2),

(35) $\quad Ru + b = \frac{1}{\Delta} \begin{bmatrix} 1 & (b+cd) \\ (a+ck) & 1 \end{bmatrix} \left\{ \begin{bmatrix} k_{2,t} \\ k_{2,t+1} \end{bmatrix} + \begin{bmatrix} \kappa E_{t-1} z_t + (a+ck)y_{t-1} \\ \kappa E_{t-1} z_{t+1} + (b+cd)y_{t+2} \end{bmatrix} \right\}$

if fiscal policy is not active. That allows us to compute the value of our policy rule, including FG elements, by comparing L_d^* and L_s^* in (19)-(20) to $L^* = \frac{1}{2}(y_t^e - \bar{y}_t)'Q(y_t^e - \bar{y}_t)$ and v^* respectively.

b) The value of FG alone. This is now easy to compute. We use (35) again, but with the forward looking elements in the policy rule cut out by setting $d=0$. This preserves the forward looking behavior in the markets (i.e. R does not become lower triangular, but is simplified) and the feed-through effects of past private expectations on y_{t+1} via the model's dynamics. But it does close down the impact of any announcements about the future on the policy process. The impact of FG policies can then be measured by comparing L_d^* and L_s^* under options a) and b).

c) Forward guidance in the absence of a (monetary) policy rule. In this case, the impact of FG (announced policy values, rather than contributions to a policy rule) can be obtained by applying (35) to L_d^* and L_s^* vs. L^* and v^* with $d=k=0$ imposed. In that exercise, $k_{2,t}$ and $k_{2,t+1}$ would be the interest rate values to be expected from rationally (optimally) chosen policies. The gains here are all from FG (if announcements are made), or from perceived/anticipated policy values (if not). To remove the anticipations effect separately (setting $b=0$) is not possible as it will take the FG impacts with it.

d) If fiscal policy is used together with monetary policy. For this case replace $k_{2,t}$ and $k_{2,t+1}$ in (35) with $(\kappa\chi + \phi)f_t$ and $(\kappa\chi + \phi)f_{t+1}$, and transfer $k_{2,t}$ and $k_{2,t+1}$ to the second vector on the right. Then compare L_d^* and L_s^* to $L^* = \frac{1}{2}(y_t^e - \bar{y}_t)'Q(y_t^e - \bar{y}_t)$ and v^* as in option a).

e) A stylized facts illustration. To illustrate the information value of FG in the presence of a known forward looking monetary policy rule, we evaluate L_d^* and L_s^* under option a). The comparators will be L^* and v^*. Under option a), the problem has been boiled down to the control of inflation ($y_t = \pi_t$) by substitution at (31). The outcomes for the output gap can then be recovered once the projections (or outcomes) for inflation are known. The policy rule (32) is known, as part of the FG

process. That yields a two period target-instrument problem – the current decision and expected outcome, and the announced policy decision and expected outcome for next period as detailed in (34) and (35). If, for simplicity, we do not discount between periods, the preference weights over two periods will be $Q = I_2$.

The rest of the model parameters are specified as follows:

$$a = \lambda = 0; \; b = (1 - \lambda)\beta + \kappa\sigma = 0.45; \; c = -\kappa\sigma = -0.55; \; \kappa = 1.1;$$

while the policy control rule (32), a prototype Taylor rule, is calibrated as $k = 1.25; \; d = 0.5$ and $k_2 = 2$ to represent, respectively, the Taylor principle, a smaller influence of future expect-ations on current decisions, and a conventional equilibrium or natural rate of interest of 2%. The model parameter calibration is justified as being mid-range estimates from the range of values given in the literature for this kind of New Keynesian or Classical monetary model (29) and (30): see Appendix C. Given that, the preference specification is $Q = I_2$ as above with desired inflation $\bar{y} = 2$ in both periods. We further assume the natural (equilibrium) rate of interest does not change so $k_{2,t-1} = k_{2,t+1} = 2$, and that the policy problem is one of wanting to escape from a period of deflation. That means output starts below trend, $z_{t-1} = -1$, with inflation also below target/equilibrium but is expected to regain equilibrium at the end of the policy interventions: $y_{t-1} = 1$ but $y_{t+1} = 2$. Finally, we assume the error process ε_t to be uncorrelated over time and independently but identically distributed, so $E\varepsilon\varepsilon' = \Sigma_1 = I_2$.

With this calibration we can go to (34) and (35) and compute

$$\Delta = 0.914, \; R = \begin{bmatrix} 1.094 & 0.191 \\ -0.731 & 1.094 \end{bmatrix} \text{ and } b = \begin{bmatrix} 3.473 \\ 1.410 \end{bmatrix}$$

and $y_t^e = 0.7$, so the comparators are $L^*=0.605$ and $v^*=1$. From there we get, using (19) from Appendix A,

$$L_d^* = \tfrac{1}{2}[E_1(b)]'[Q - QR(R'QR)^{-1}R'Q][E_1(b)] = 0.0053$$

and $\quad L_s^* = tr[\text{I-R}(R'\text{R})^{-1}R']Q[\text{I-R}(R'\text{R})^{-1}R'] = tr[\text{I} - R(R'\text{R})^{-1}R'] = 2 - 2 = 0$

using symmetry, idempotency and $Q=I$.

These results imply a 99% improvement in the deterministic (controllability) part of the policy problem, and a complete resolution of the stabilization part (perfect stabilisability) over the status quo ante of doing nothing, as a result of using FG. Nevertheless gains of this size should not come as a surprise. This is a one target, one instrument policy problem set over two periods with a model that is dynamically controllable, and hence stabilisable, over a single period. So such gains

are just what a formal analysis says should happen (Hughes Hallett and Acocella 2016a,b respectively). If there were more targets, and hence a longer lead time to gain controllability, the gains would be smaller and more difficult to achieve. The point is that they have been achieved here by FG acting as a surrogate instrument; and given that the policymakers' ideal values will predictably be achieved, there is no point in expecting time inconsistency to arise.

Appendix C: Parameter Estimates from the Literature

Most models are estimated in a form similar to (29) and (30). For example:

$$z_t = a_1 E_t z_{t+1} + (1 - a_1) z_{t-1} + a_2 (r_t - E_t \pi_{t+1}) + \varepsilon_t \qquad (36)$$

and

$$\pi_t = b_1 E_t \pi_{t+1} + (1 - b_1) \pi_{t-1} + b_2 y_t + \eta_t \qquad (37)$$

where, in the notation of (29) and (30), the following transformations hold:

$a = \lambda = 1 - b_1 = 0$ if $b_1 = 1.0$;

$b = (1 - \lambda)\beta + \kappa\sigma = b_1 + b_2 a_2 = -0.45$ if $b_2 = 1.1$ and $a_2 = -0.5$ (implying $\beta = 1$);

$\kappa = b_2 = 1.1$; and $c = -\kappa\sigma = b_2 a_2 = -0.55$ (implying $\sigma = 0.5$).

These calibrated values for the b_1, b_2 and a_2 coefficients in (36) and (37) above can be justified by the estimated values available in the literature:

i) a_2 = -0.002 (Dennis (2004)); -0.015 to -0.023 (Fuhrer (2015)); -0.27 to -0.59 (Dées, Pesaran, Smith, and Smith (2010))
ii) b_1 = 0.5 (Dennis (2004)); 0.61to 0.72 (Gali, Gertler, and Lopez-Salido (2005)); 0.98 (Schorfheide (2008)); 0.91 (Dées, Pesaran, Smith, and Smith (2010))
iii) b_2 = 0.4 (Dennis (2004)); 0.14 (Gali, Gertler, and Lopez-Salido (2005)); 0.4 (Schorfheide (2008)); 0.13 to 0.16 (Dées, Pesaran, Smith, and Smith (2010))

Appendix D: Alternative Interpretations of the Theoretical Model

To underpin the generality of our results, there are several different ways to justify the use of the model in equations (29)-(30). They are all taken from Gali's (2015) review of models suitable for studying monetary policy and business cycles.

1. Justifying (29)-(30) on a Classical Monetary theory basis:
a. The aggregate demand (IS) relation in equation (29) corresponds exactly with the equilibrium path for output in Gali (2015), equations (9) and (16) on pp. 21-23: with either i) no exogenous preference shifts between consumption and employment [equivalent to saying we have stable labor preferences and a constant marginal utility of consumption]; or ii) no persistence in those preference shifts; and iii) no discounting by households, or very little; and iv) a degree of output or consumption smoothing that generalizes the traditional monetary theory specification with either $\beta = 0$ or $\lambda = 1$, to the possibility that neither is true. Obviously we have taken the second possibility. Note also that, within this specification, $a_2 = -(1/\sigma)$ where σ is the degree of

curvature in the utility function, itself a reflection of the degree of risk aversion in consumption and hence output, which in turn justifies introducing output smoothing [$\beta \neq 0, \lambda \neq 1$]. Thus, to expect $a_2 \leq 0$ to be small is to expect $\sigma > 0$ and to be larger; that is to say a substantial degree of risk aversion, which again justifies including an output or consumption smoothing term.

b. Aggregate supply (Phillips curve) relation in equation (30): this is implied by the price level equation on p. 26, Gali (2015), with the steady state inflation term (or inflation target) replaced by an estimated inflation trend and with the previous real monetary shock determining the current output gap deviations [or, equivalently, previously expected inflation deviations determining the current output gap].

2. An extended monetary theory version of the model:

Although the description above represents the standard version of the classical monetary theory model, Gali (2015) points out that the aggregate demand equation (29) also follows from a more sophisticated version that allows for non-separable utilities between consumption, employment, prices and money; and real where wages may change in response to changes in nominal interest or policy rates [Gali (2015), p. 39]. In this version, we get (29) if the elasticity of real wages to changes in nominal interest rates is zero. This implies changes in the marginal utility of consumption falls by the same amount as any rise in interest rates, because the quantity of labor supply will fall by a matching amount. The result is of course that equilibrium output becomes invariant to monetary policy, which is just a statement of monetary neutrality (a property we would expect from any monetary theory model).

3. Equations (29)-(30) on a New Keynesian Basis

a. Aggregate demand (IS), equation (29): This is simply equation (23), p. 63, of Gali (2015) with output smoothing to replace the next period's expectation for the output gap – as in the classical monetary model above – and the constant $(1/\sigma)r^n$ ignored [alternatively $E_t(\varepsilon_t) = r^n/\sigma$]. Once again $a_2 = -(1/\sigma)$ where σ is the degree of curvature/risk aversion in the utility function justifying the introduction of output or consumption smoothing.

b. Aggregate supply (Phillips curve), equation (30): This is equation (22) in Gali (2015), p. 63, with inflation smoothing and $b_2 = \lambda \left[\sigma + \frac{\phi + \alpha}{1 - \alpha} \right]$, where λ measures the impact of price stickiness or imperfect competition on inflation; α the decreasing returns to labor in production; and ϕ is the impact of employment on real wages (equivalently the utility elasticity with respect to employment, or disutility of extra work).